MUCH ADO ABOUT N=2

30 YEARS OF SUPERGRAVITY
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The first 15 years ....

N=2 supersymmetry and supergravity

- Fayet: *Fermi-Bose Hypersymmetry*, Nucl Phys. B113 (1976)
- Ferrara, van Nieuwenhuizen: *Consistent Supergravity with complex spin 3/2 Gauge Fields*, Phys. Rev. Lett. 37 (1976)

gauged N=2 supergravity

- Freedman, Das: *Gauge Internal Symmetry In Extended Supergravity*, Nucl. Phys. B120 (1977)

N=2 supersymmetric gauge theory

- Grimm, Sohnius, Wess: *Extended Supersymmetry and Gauge Theories*, Nucl. Phys. B133 (1978)

off-shell N=2 supergravity

- Fradkin, Vasiliev: *Minimal Set of Auxiliary Fields* in SO(2) Extended Supergravity, Phys. Lett. B85 (1979)
- de Wit, van Holten: *Multiplets of Linearized SO(2) Supergravity*, Nucl. Phys. B155 (1979)

matter couplings, gaugings

- **de Wit, Van Proeyen:** *Potentials and Symmetries of General Gauged N=2 Supergravity - Yang-Mills Models*, Nucl. Phys. B245 (1984)
- **de Wit, Lauwers, Van Proeyen:** *Lagrangians of N=2 Supergravity - Matter Systems*, Nucl. Phys. B255 (1985)
- **D'Auria, Ferrara, Fré:** *Special and quaternionic isometries: General couplings in N=2 supergravity and the scalar potential*, Nucl. Phys. B359 (1991)

c-map, electric/magnetic duality

- **Cecotti, Ferrara, Girardello:** *Geometry of Type II Superstrings and the Moduli of Superconformal Field Theories*, Int. J. Mod. Phys. A4 (1989)

special geometry and Calabi-Yau three-folds

- **Strominger:** *Special Geometry*, Commun. Math. Phys. 133 (1990)
- **Candelas, de la Ossa:** *Moduli Space of Calabi-Yau Manifolds*, Nucl. Phys. B355 (1991)

MANY MORE CONTRIBUTIONS: 481.000 GOOGLE HITS !!

Next 15 years: new perspectives

- Supersymmetric gauge theories Seiberg, Witten, 1994
- Topological theories Witten, 1991

  topological strings Bershadsky, Cecotti, Ooguri, Vafa, 1993
- Black holes Ferrara, Kallosh, Strominger, 1995
- String effective actions: flux compactifications, gaugings

N=2 supersymmetric actions
moduli stabilization
supersymmetry breaking

General features

- 8 supersymmetries: N=2 in four space-time dimensions
- R-symmetry: $\mathrm{SU}(2) \times \mathrm{U}(1)$ [4D]
- Spherically symmetric BPS states: black holes, magnetic monopoles, dyons
- Off shell formulations

the latter is especially relevant when dealing with higher-derivative couplings

N=2 supermultiplets

vector supermultiplet $(\underset{2}{X}, \underset{8}{\Omega^i}, \underset{3}{F_{\mu\nu}}, \underset{3}{Y^{ij}})$ reduced chiral supermultiplet

tensor supermultiplet $(\underset{3}{L^{ij}}, \underset{8}{\varphi^i}, \underset{3}{E^\mu}, \underset{2}{G})$

hypermultiplet $(\underset{4}{\phi}, \underset{8}{\zeta^\alpha})$ no finite off-shell realization

Weyl supermultiplet $(\underset{6}{T_{ab}{}^{ij}}, \underset{16}{\psi_\mu{}^i}, \underset{8}{\chi^i}, \underset{4\times 3}{F_{\mu\nu}}, \underset{5}{R_{\mu\nu}{}^{ab}}, \underset{1}{D})$

gravitini

$\mathrm{SU}(2) \times \mathrm{U}(1)$
R symmetry

graviton

off-shell: supersymmetry transformations are independent of the Lagrangian

SUPERCONFORMAL MULTIPLET CALCULUS

consider matter supermultiplets in a
superconformal supergravity background

two examples:

◆ massive gauge fields: Stuckelberg

matter multiplet in a gauge field background

$$\mathcal{L} = -\tfrac{1}{4}(\partial_\mu V_\nu - \partial_\nu V_\mu)^2 - \tfrac{1}{2}M^2|(\partial_\mu - \mathrm{i}V_\mu)\mathrm{e}^{\mathrm{i}\phi}|^2$$

$$\longleftrightarrow \quad -\tfrac{1}{4}(\partial_\mu V_\nu - \partial_\nu V_\mu)^2 - \tfrac{1}{2}M^2{V_\mu}^2$$

◆ gravity: Weyl

matter multiplet in a conformal gravity background

$$\mathcal{L} = \sqrt{g}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - \tfrac{1}{6}\sqrt{g}R\,\phi^2 \quad \longleftrightarrow \quad -\frac{1}{2\kappa^2}\sqrt{g}R$$

GAUGE EQUIVALENCE

building block procedure: irreducibility

Combine the two examples which involve the modulus and the phase of a scalar field

$$\mathcal{L} = \sqrt{g}\,g^{\mu\nu}[(\partial_\mu - \mathrm{i}A_\mu)\bar{X}][(\partial_\nu + \mathrm{i}A_\nu)X] - \tfrac{1}{6}\sqrt{g}R\,|X|^2$$

X is projectively defined: $X \leftrightarrow z\,X$ are identified

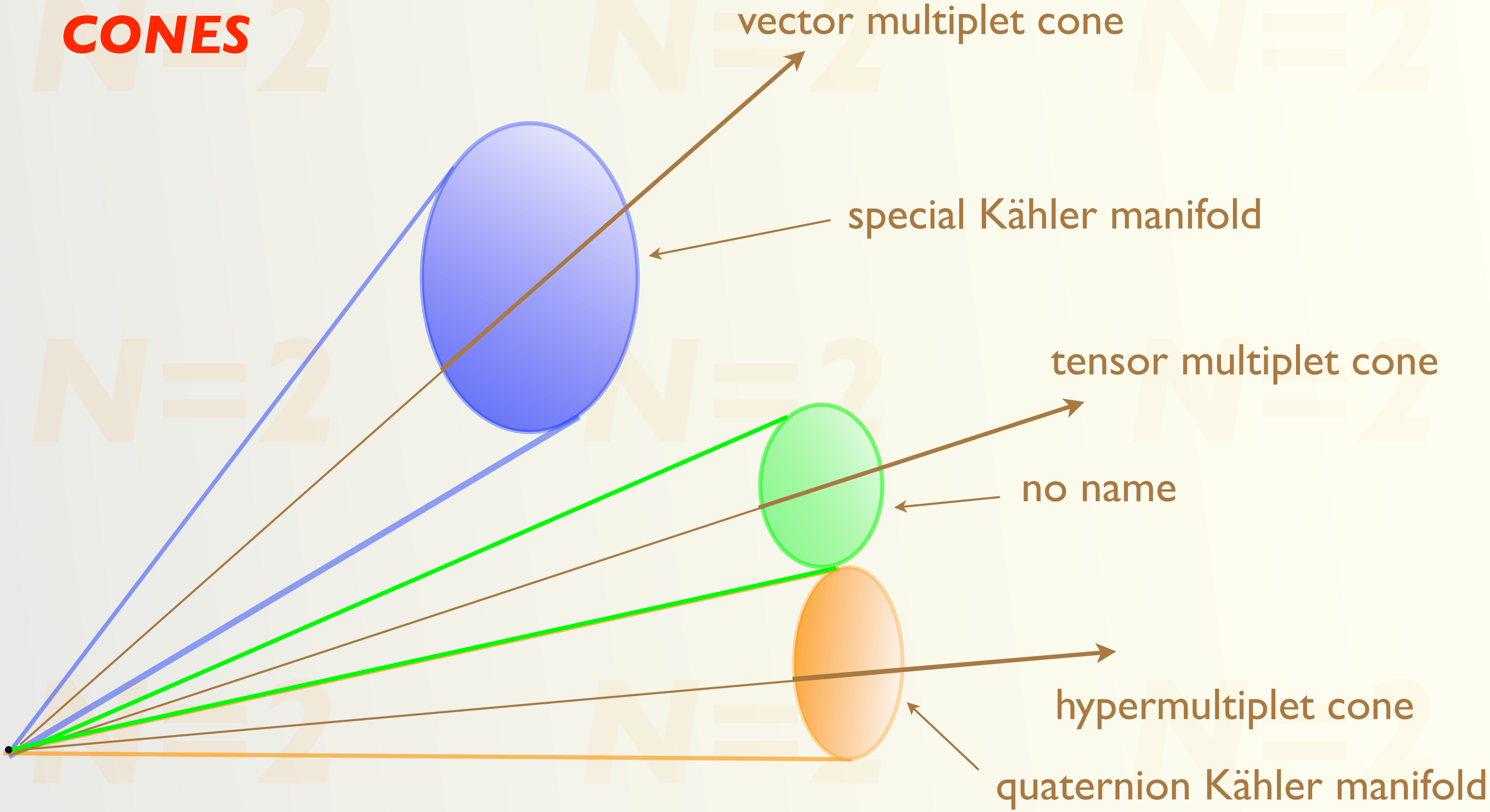
Taking several supermultiplets with corresponding scalars $X^\Lambda\,,\quad \Lambda = 0, 1, \ldots, n\,,$ leads to an n-dimensional special Kähler geometry.

The X^Λ parametrize a cone over the special Kähler space. The latter arises as the result of taking a superconformal quotient.

This generalizes to all supermultiplets !

Generally: Life on the cone is simpler !!

CONES



tensor-scalar duality: the hypermultiplet and tensor multiplet cones are not independent

non-renormalization theorem: the vector multiplet cone is decoupled from the other two (at two-derivative level)

Coupling to supergravity is based on 'potentials' such as the Kähler and hyperkähler potentials. These are homogeneous functions invariant under R symmetry

for tensor, hyper and vector multiplet cones :

$$\begin{aligned}
\chi_{\text{tensor}}(L) &= 2\,F_{IJ}(L)\,L_{ij}{}^{I}\,L^{ij\,J} \\
\chi_{\text{hyper}}(\phi) &= \tfrac{1}{2}\varepsilon^{ij}\bar{\Omega}_{\alpha\beta}\,A_i{}^{\alpha}(\phi)A_j{}^{\beta}(\phi) \\
\chi_{\text{vector}}(X,\bar{X}) &= \mathrm{i}\,\big(X^{\Lambda}\bar{F}_{\Lambda} - \bar{X}^{\Lambda}F_{\Lambda}\big) = N_{\Lambda\Sigma}\,X^{\Lambda}\bar{X}^{\Sigma}
\end{aligned}$$

where $F_{\Lambda} = \partial F(X)/\partial X^{\Lambda}$
and the function $F(X)$ holomorphic and homogeneous

These three potentials encode the most general matter couplings to supergravity with tensor, hyper and vector multiplets

dW, Van Proeyen, 1984
dW, Kleijn, Vandoren, 1999
dW, Rocek, Vandoren, 2001
dW, Saueressig, 2006

Deformations (gaugings)

or: what is the most 'general' N=2 supergravity ?

tensor multiplets: no gauging seems possible.
hypermultiplets: symmetries most transparantly realized on the hyperkähler cone by tri-holomorphic isometries.
vector multiplets: must comprise some super-Yang-Mills theory. Subtle because of electric/magnetic duality !

The gauge group must be embedded into the rigid invariance group. For N=2 systems, this is a product group, in view of the fact that vector multiplets and hypermultiplets do not couple directly. For the vector multiplets the rigid invariance group may be realized through electric/magnetic dualities.

Involves the DUAL field strength: $G_{\mu\nu\,\Lambda} \propto \sqrt{|g|}\,\varepsilon_{\mu\nu\rho\sigma}\,\frac{\partial\mathcal{L}}{\partial F_{\rho\sigma}{}^{\Lambda}}$

Invariance of field equations and Bianchi identities !

Electric/magnetic duality transformations:

$$\begin{pmatrix} F^\Lambda \\ G_\Lambda \end{pmatrix} \longrightarrow \begin{pmatrix} U^\Lambda{}_\Sigma & Z^{\Lambda\Sigma} \\ W_{\Lambda\Sigma} & V_\Lambda{}^\Sigma \end{pmatrix} \begin{pmatrix} F^\Sigma \\ G_\Sigma \end{pmatrix}$$

$\in \mathrm{Sp}(2n+2)$ Gaillard, Zumino, 1981

Likewise: $X^M = (X^\Lambda, F_\Lambda)$ defines an $\mathrm{Sp}(2n+2)$ vector.

dW, Van Proeyen, 1984

The gauge group generators: $\delta X^M = -\Lambda^P\, T_{PN}{}^M\, X^N$

Subject to constraint $T_{(MN}{}^Q \Omega_{P)Q} = 0$

Standard lore: one must choose an e/m duality frame where charges are electric:

$$Z^{\Lambda\Sigma} = 0\,, \qquad U^\Lambda{}_\Gamma V_\Sigma{}^\Gamma = \delta^\Lambda{}_\Sigma$$

structure constants

$$\delta X^\Lambda = -\Lambda^\Sigma T_{\Sigma\Gamma}{}^\Lambda\, X^\Gamma \qquad \delta F(X) = -\tfrac{1}{2}\Lambda^\Sigma\, T_{\Sigma\Lambda\Gamma}\, X^\Lambda X^\Gamma$$

not invariant !

requires extra CS-like term

$$\mathcal{L}_{\text{extra}} \propto \varepsilon^{\mu\nu\rho\sigma} T_{\Lambda\Sigma\Gamma} A_\mu{}^\Lambda A_\nu{}^\Sigma (\partial_\rho A_\sigma{}^\Gamma + \tfrac{3}{8} T_{\Xi\Delta}{}^\Gamma A_\rho{}^\Xi A_\sigma{}^\Delta)$$

dW, Lauwers, Van Proeyen, 1985

However: also magnetic charges can be incorporated !

This requires tensor fields BUT the
scalar potential is insensitive to that !
Additional CS-like terms involving
magnetic gauge fields.

dW, Samtleben, Trigiante, 2005
Louis, Micu, 2002
Sommovigo, Vaula, 2004

$\mathrm{U}(1)$ moment map: $\nu_M \propto T_{MN}{}^Q\,\Omega_{PQ}\bar{X}^N X^P$

Gauge invariance: $T_{MN}{}^Q\,\Omega_{PQ}X^N X^P = 0$

Vector multiplet scalar potential (valid in any e/m frame):

$$\begin{aligned}\mathcal{V} &\propto \mathrm{i}\,\Omega_{MN}\,(\bar{X}^P T_{PR}{}^M X^R)(X^Q T_{QS}{}^N \bar{X}^S)\\ &\propto \mathrm{i}(\bar{F}-F)_{\Lambda\Sigma}(\bar{X}^P T_{PR}{}^\Lambda X^R)(X^Q T_{QS}{}^\Sigma \bar{X}^S)\end{aligned}$$

dW, de Vroome, in preparation

Higher-order derivative couplings

There are crucial changes in the presence of higher-order derivatives !

For instance consider N=2 supersymmetric gauge theories

Kähler potential: $K(X,\bar{X}) \propto \mathrm{i}\bar{X}^{\Lambda}F_{\Lambda}(X) - \mathrm{i}X^{\Lambda}\bar{F}_{\Lambda}(\bar{X})$

Non-abelian one-loop corrections are inconsistent with this special geometry parametrization. In fact, these corrections are part of an independent supersymmetric invariant whose leading term involves terms quartic in the field strengths!

dW, Grisaru, Rocek, 1996

Also for tensor multiplets higher-order derivative couplings have been constructed

dW, Saueressig, 2006

No vector-hyper non-renormalization theorem anymore

Gravitational higher-order derivative corrections are relevant for the subleading contributions to the black hole entropy in the limit of large charges

chiral class: Weyl background $F(Y) \longrightarrow F(Y, \Upsilon)$

$\uparrow$ Weyl background

$Y^\Lambda - \bar{Y}^\Lambda = \mathrm{i}p^\Lambda$ magnetic charges

$F_\Lambda - \bar{F}_\Lambda = \mathrm{i}q_\Lambda$ electric charges

$\Upsilon = -64$

NOTE : $Y^\Lambda + \bar{Y}^\Lambda$ and $F_\Lambda + \bar{F}_\Lambda$ play the role of electro- and magnetostatic potentials

Attractor equations remain valid in the presence of Υ!

Entropy formula:

$$\mathcal{S}_{\rm macro}(p,q) = \pi\,\Sigma\Big|_{\rm attractor} = \pi\Big[|Z|^2 - 256\,{\rm Im}\,F_\Upsilon\Big]_{\Upsilon=-64}$$

$|Z|^2 = p^I F_I - q_I Y^I$

Cardoso, dW, Mohaupt, 1998

based on the Wald entropy based on a conserved Noether potential

Wald, 1993

In one particular case:

$$S_{\rm macro} = 2\pi\sqrt{\frac{1}{6}|\hat{q}_0|\,(C_{ABC}\,p^A p^B p^C + c_{2A}\,p^A)}$$

agrees with the result of microstate counting for both leading and subleading contributions of a five-brane wrapped around a CY 4-cycle and compactified on an extra S^1

Maldacena, Strominger, Witten, 1997

Also applications to N=4 !

N=2 SUPERGRAVITY IS ALIVE AND KICKING

◆ conformal multiplet calculus leads to cones. For the cones many symmetry properties are much more transparant

◆ these cones are described in terms of certain functions/potentials such as a special Kähler potentials or hyperkähler potentials. These fully encode the corresponding supergravity theory

◆ gaugings can be written down irrespective of the electric/magnetic duality frame

◆ off-shell representations are crucial for higher-derivative couplings, many of which have been constructed recently

◆ many important applications ranging from gauge theories, effective string theory actions, to black holes