Introduction to the

Superfield formalism.

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N=1 Supersymmetry:

$$\{ Q_\alpha , \bar{Q}_{\dot\beta} \}_+ = 2\sigma^m_{\alpha\dot\beta} P_m$$

Q_α, $\bar{Q}_{\dot\alpha}$: twocomponent Weyl spinors

Bookkeeping parameters

$$\left.\begin{array}{l} \xi^\alpha, \eta^\alpha, \theta^\alpha \ldots \\ \bar\xi_{\dot\alpha}, \bar\eta_{\dot\alpha}, \bar\theta_{\dot\alpha} \ldots \end{array}\right\} \text{anticommuting spinors}$$

$$\xi Q = \xi^\alpha Q_\alpha \quad , \quad \bar\xi \bar{Q} = \bar\xi_{\dot\alpha} \bar{Q}^{\dot\alpha}$$

$$[\xi Q , \bar\xi \bar Q] = 2 (\xi \sigma^m \bar\xi) P_m$$

Jacoby identity:

$$[\xi\chi[\eta\psi,\rho\phi]] + [\eta\psi[\rho\phi,\xi\chi]] + [\rho\phi[\xi\chi,\eta\psi]] = 0$$

Wess, Zumino

Salam, Strathdee

Zumino, Ferrara

"Group" parameters: $x^m, \theta_\alpha, \bar{\theta}^{\dot{\alpha}}$

$$G(x,\theta,\bar{\theta}) = \exp i\{-x^m P_m + \theta Q + \bar{\theta}\bar{Q}\}$$

multiplication (Hausdorff formula)

$$G(0,\xi,\bar{\xi})\cdot G(x,\theta,\bar{\theta}) =$$

$$= G(x + i\theta\sigma\bar{\xi} - i\xi\sigma\bar{\theta},\ \theta+\xi,\ \bar{\theta}+\bar{\xi})$$

$$\equiv G(x', \theta', \bar{\theta}')$$

motion in parameter space

$$\{x,\theta,\bar{\theta}\} \to \{x',\theta',\bar{\theta}'\}$$

parameter space

$\Rightarrow$ Superspace

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motion generated by (left motion)
differential operators in superspace:

$$\xi Q + \bar{\xi}\bar{Q} = \xi\left(\frac{\partial}{\partial\theta} - i\sigma^m\bar{\theta}\partial_m\right) + \bar{\xi}\left(\frac{\partial}{\partial\bar{\theta}} - i\theta\sigma^m\partial_m\right)$$

$$\underline{\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2i\sigma^m_{\alpha\dot{\alpha}}\partial_m}$$

right motion:

$$\xi D + \bar{\xi}\bar{D}$$

$$\underline{\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i\sigma^m_{\alpha\dot{\alpha}}\partial_m}$$

$$\{D, Q\}_+ = 0$$

<u>Super field:</u>

16 = 16

$$f(x) \sim F(x, \theta, \bar{\theta})$$

$$= f(x) + \theta\varphi(x) + \bar{\theta}\bar{\chi}(x)$$
$$+ \theta\theta\, m(x) + \bar{\theta}\bar{\theta}\, n(x) + \theta\sigma^m\bar{\theta}\, v_m(x)$$
$$+ \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\psi(x) + \theta\theta\bar{\theta}\bar{\theta}\, d(x)$$

$$\delta_\xi F = (\delta_\xi f)(x) + \theta(\delta_\xi \varphi)(x) + \cdots$$
$$+ \cdots$$

$$\delta_\xi F = \xi Q \times F + \bar{\xi}\bar{Q} \times F$$

<u>differential operators</u>

Component fields form a multiplet.

In general reducible

Reduction of the multiplets

$$\bar{D}_{\dot\alpha}\, \phi(x, \theta, \bar\theta) = 0$$

$$\{\bar{D}, Q\} = 0 \qquad \{\bar{D}, \bar{Q}\} = 0$$

$\phi(x, \theta, \bar\theta)$: Scalar multiplet

$$\phi = A(y) + \sqrt{2}\,\theta\psi(y) + \theta\theta F(y) \qquad 4 = 4$$

$$y = x + i\theta\sigma\bar\theta$$

Component fields

$$\delta_\xi A = \sqrt{2}\,\xi\psi$$

$$\delta_\xi \psi = i\sqrt{2}\,\sigma^m \bar\xi\, \partial_m A + \sqrt{2}\,\xi F$$

$$\delta_\xi F = i\sqrt{2}\,\bar\xi\bar\sigma^m \partial_m \psi$$

vector multiplet :

$$V^{+} = V \qquad 8 = 8$$

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Superfields $\to$ component fields

component fields $\longrightarrow$ superfields

$\delta_\xi \, C(x) = \dots$

$$\tilde{C}(x, \theta, \bar\theta) = e^{(\theta\delta_\xi + \bar\theta\delta_{\bar\xi})} \cdot C(x)$$

$$\delta_\xi \, \tilde{C} = \Big(\underbrace{\xi Q + \bar\xi \bar Q}_{\text{differential operator.}}\Big) \times \tilde{C}$$

highest component will always transform into a space derivative of lower components.

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Reduction of the product of multiplets

$\phi \cdot \phi \quad : \quad \bar{D}(\phi \cdot \phi) = 0$

is a scalar multiplet again:

$$\phi_i \phi_j = A_i(y) A_j(y) + \sqrt{2}\,\theta\left(\psi_i A_j + A_i \psi_j\right)$$
$$+ \theta\theta\left(A_i F_j + A_j F_i - \psi_i \psi_j\right)$$

$\phi^+ \phi \quad$: is a vector multiplet

$$\phi_i^+ \phi_j = \ldots$$

$$\ldots$$
$$\theta\theta\bar{\theta}\bar{\theta}\left\{ F_i^* F_j + \frac{1}{4} A_i^* \Box A_j + \frac{1}{4}(\Box A_i^*) A_j \right.$$
$$- \frac{1}{2}\partial_m A_i^* \partial^m A_j$$
$$\left. + \frac{i}{2}\partial_m \bar{\psi}_i \bar{\sigma}^m \psi_j - \frac{i}{2}\bar{\psi}_i \bar{\sigma}^m \partial_m \psi \right\}$$

$$\mathcal{L} = \phi_i^+ \phi_i \Big|_{\theta^2\bar{\theta}^2 \text{ comp}} + \left[\left(\frac{1}{2} m_{ij}\, \phi_i \phi_j + \frac{1}{3} g_{ijk} \phi_i \phi_j \phi_k\right)\Big|_{\theta^2} + h.c.\right]$$

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Superfield propagators:

Fujikawa Lang.

$$\langle 0| T\{\phi(x,\theta,\bar\theta)\,\phi(x',\theta',\bar\theta')\}|0\rangle$$

$$= -im\,\delta(\theta-\theta')\exp[i(\theta\sigma\bar\theta-\theta'\sigma\bar\theta')\partial]$$

$$\cdot\Delta_F(x-x')$$

$$\langle 0| T\{\phi\ \ \phi^+\}|0\rangle =$$

$$= i\exp[i(\theta\sigma\bar\theta+\theta'\sigma\bar\theta'-2\theta\sigma^m\bar\theta')\partial]\cdot\Delta_F$$

Integration

$$\eta : \int d\eta\, f(x)=0 \qquad \int d\eta\,\eta = 1$$

$$F(x,\eta) = f(x)+\Delta(x)\eta$$

$$\int F(x,\eta) = \Delta(x) = F(x,0)$$

$$\delta(\eta)=\eta$$

$$\delta(\eta)\delta(\eta)=0$$

combinatorial reason for the cancellation of divergencies.

Feynman rules for superfields:

$$\langle 0 | T\{\phi(z) \cdots \phi^+(z')\} 0 \rangle$$

$$= \langle 0 | T\{\phi^0(z) \cdots \phi^{0+}(z')\, e^{i\int \mathcal{L}^0_{int}}\} | 0 \rangle$$

Wick ordering → Feynman rules:

$$\delta(\theta - \theta')^2 = 0 \;!$$

All diagrams with external ϕ <u>or</u> ϕ^+ legs only vanish

Interaction: ϕ^3

Mass ϕ^2

→ Non renormalization theorem

Grisaru, Roček, Siegel

Gauge theories:

gauge transformation:

$$\phi' = e^{-i\Lambda}\phi \qquad \bar{D}\Lambda = 0 !$$

interaction $g_{ijk}\phi_i\phi_j\phi_k$

$$\phi^{+\prime}\phi' = \phi^{+} e^{i\Lambda^{+}} e^{-i\Lambda}\phi$$

Connection:

$$e^{V'} = e^{-i\Lambda^{+}} e^{V} e^{+i\Lambda}$$

$$\phi^{+\prime} e^{V'} \phi' = \phi^{+} e^{V} \phi$$

leads to minimal gauge coupling.

Cinetic part for the Vectorfield

$V: \ 8 = 8$

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}\, e^{-V} D_\alpha e^{V}$$

$$W_\alpha' = e^{-i\Lambda} W_\alpha\, e^{i\Lambda}$$

$$\bar{D} W_\alpha = 0$$

$$\mathcal{L} = \frac{1}{4} Tr\left(W^\alpha W_\alpha \Big|_{\theta\theta} + \bar{W}_{\dot\alpha} \bar{W}^{\dot\alpha} \Big|_{\bar\theta\bar\theta} \right)$$

$$= \frac{1}{2} D^2 - \frac{Tr}{4} F^{mn} F_{mn} - i\lambda\sigma\partial\bar{\lambda}$$

$4 = 4$

Superspace – a playground
for differential geometry

manifold:

$$z^M = (x^m, \theta^\mu, \bar\theta_{\dot\mu})$$

$$z^M z^N = (-1)^{mn} z^N z^M$$

forms:

$$dz^M \sim (dx^m, d\theta^\mu, d\bar\theta_{\dot\mu})$$

$$dz^M dz^N = -(-)^{mn} z^N, z^M$$

p-forms:

Exterior derivative:

$$d : \Omega^p \rightarrow \Omega^{p+1}$$

$$d(\Omega + \Xi) = d\Omega + d\Xi$$

$$d(\Omega, \Xi) = \Omega\, d\Xi + (-1)^q\, d\Omega\, \Xi$$

$$d \cdot d = 0$$

Frame: $E^A = dZ^M E_M{}^A$

Flat superspace:

$E^A = dZ^M e_M{}^A$

such that

$dZ^M \partial_M = e^A D_A = e^a \partial_a + e^\alpha D_\alpha + e_{\dot\alpha} \bar{D}^{\dot\alpha}$

Torsion:

$de^a = -2i e^\alpha \sigma^a_{\alpha\dot\alpha} e^{\dot\alpha}$

$de^\alpha = 0 \, , \; de_{\dot\alpha} = 0$

Flat superspace has Torsion!

Lie group:

0-form (fields)

$$\delta_\lambda A_r = -i\lambda^+_{rs} T^t_s$$

gauge Transformations.

Lie algebra valued vector superfield: ϕ: one form

Fieldstrength (Curvature)

$$F = d\phi + \phi\cdot\phi = \frac{1}{2} e^A e^B F_{BA}$$

Constraints:

$$F_{\alpha\beta} = F_{\dot\alpha\dot\beta} = F_{\alpha\dot\beta} = 0$$

Bianchi identities: $\mathcal{D}F = 0$

$$\rightarrow F_{a\alpha} = i\sigma_{a\alpha\dot k} \bar W^{\dot k}$$

$$\bar{\mathcal{D}} W = 0 \quad \mathcal{D}\bar W = 0$$

$$\mathcal{L} = W^\alpha W_\alpha + \bar W_{\dot\imath} \bar W^{\dot\imath}$$

gravity Deformation