N=2 Supergravity *D*-terms, Cosmic Strings and Brane Cosmologies

Antoine Van Proeyen

K.U. Leuven

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Supergravity for string cosmology

- Since a few years there are models for cosmology in string theory
- The effective field theories are supergravity theories. Helps to maintain technical control
- An important issue is the ‘uplifting’:
  terms that add vacuum energy.
- These are provided by *D*-terms, which may include Fayet-Iliopoulos (FI) terms
- Consistency issues in supergravity.

Plan

1. Supergravity *D*-terms and *R*-symmetry.
2. Cosmic strings: energy described by a FI term
3. *D*-term inflation: a consistency problem
4. KKLT (modified to D3/D7)
   How do the *D*-terms and superpotential respect the consistency conditions ?
5. *N*=2 *D*-terms: consistency requirements and models related to *N*=1.

1. Supergravity *D*-terms and *R*-symmetry.

- *R*-symmetry
- Recapitulation of supergravity ingredients
- *D*-terms, including FI terms, and gauge symmetries

R-symmetry

- R-symmetry is the algebra that rotates the supersymmetries: $[R_A, Q^i] = (t_A)^i{}_j Q^j$
  In *N=1* : $[R, Q] = \mathrm{i}\gamma_5 Q$
- Therefore different members of a multiplet have different transformation laws

useful for model building, see talks P. Fayet, P. Nilles, …

- Compare: gauge symmetries (of vectors in vector multiplets) commute with the supersymmetries.
- But this is more complicated in supergravity

see talk J.P. Derendinger

- Appears in the superconformal , super-AdS and super-dS algebras.

Superalgebras and R-symmetry

- Semisimple superalgebras for spacetime symmetries have a bosonic subalgebra of the form:

  spacetime algebra × R-symmetry

Nahm, 1978

	spacetime algebra	superalgebra	R-symmetry
Conformal	so(4,2) = su(2,2)	su(2,2|N)	U(N)
AdS	so(3,2) = usp(4)	osp(N|4)	SO(N)
dS	so(4,1) = usp(2,2)	$osp(N^*\|2,2)$	$SO(N^*)$

N even

Non-compact or absent (N=1) for de Sitter
→ no vacua with positive kinetic energies

Superconformal: U(1) for N=1 and U(1)× SU(2) for N=2

N=1 supergravity

see talk B. de Wit

- Start from superconformal multiplets. Their structure is analogous to rigid susy
- Superconformal gauge fields: e_μ^a, ψ_μ, A_μ
- Chiral multiplets X^I, χ^I, h^I
- Vector multiplets (in WZ gauge) $W_\mu^\alpha, \lambda^\alpha, D^\alpha$
- Actions built from F- and D-terms

$$\mathcal{L} = [\mathcal{N}(X,\bar{X})]_D + [\mathcal{W}(X)]_F + \left[f_{\alpha\beta}(X)\bar{\lambda}_L^\alpha \lambda_L^\beta\right]_F$$

Restrictions to be conformal invariant.

Conformal methods: Kaku, Townsend, van Nieuwenhuizen, Ferrara, de Wit; general actions: Cremmer, Julia, Scherk, Ferrara, Girardello, van Nieuwenhuizen, AVP

Superconformal parametrization

- Convenient to parametrize: $X^I = \{Y, z^i\}$ $\quad \begin{array}{rcl} I &=& 0,1,\ldots,n \\ i &=& 1,\ldots,n \end{array}$

$$\delta Y = \Lambda_D Y - \tfrac{1}{3}\mathrm{i}\Lambda_{\mathrm{U}(1)} Y$$

In action: $\mathcal{L} = [\mathcal{N}(X,\bar{X})]_D + [\mathcal{W}(X)]_F + \left[f_{\alpha\beta}(X)\bar{\lambda}_L^\alpha \lambda_L^\beta\right]_F$

we should have (for conformal inv.) $\quad \mathcal{N} = -3Y\bar{Y}\mathrm{e}^{-K(z,\bar{z})/(3M_P^2)}$

$$\mathcal{W} = Y^3 W(z)$$

- Dilatational gauge fixing: $Y\bar{Y}\mathrm{e}^{-K(z,\bar{z})/(3M_P^2)} = M_P^2$
- U(1) gauge fixing: $Y = \bar{Y}$

Is ϕ^3 of Mark Grisaru's talk

N=1 Potential

- General structure of potential in supergravity:

V = Σ_{fermions} (δ fermion) (metric) (δ fermion)

$$V = h^I G_{I\bar{J}} \bar{h}^{\bar{J}} + \tfrac{1}{2} D^\alpha (\mathrm{Re}\, f_{\alpha\beta}) D^\beta = V_F + V_D$$

$$G_{I\bar{J}} = -\partial_I \partial_{\bar{J}} \mathcal{N}$$ Signature: $(- + \ldots +)$

due to the fact that the conformal action for a scalar is $\mathcal{L} = -\sqrt{g}\phi\Box\phi + \frac{1}{6}\sqrt{g}R\phi^2$

pos. kin. energy for gravity
implies neg.kin.energy for scalar

$$X^I = \{Y, z^i\}$$

$$V_F = \mathrm{e}^{K/M_P^2}\left[-3M_P^2 W\bar{W} + (D_i W) g^{i\bar{j}} (D_{\bar{j}}\bar{W})\right], \qquad D_i W = \partial_i W + M_P^{-2}(\partial_i K) W$$

Gauge symmetries and *R*-symmetry

- Gauge symmetries of vectors commute with susy in rigid supersymmetric theory.
- Compensating field *Y* may also transform:

$$\delta_G Y = Y r_\alpha(z) \Lambda^\alpha, \qquad \delta_G z^i = k_\alpha^i(z)\Lambda^\alpha$$

- FI term is a phase transformation: $r_\alpha = \frac{1}{3} \mathrm{i} g \xi_\alpha M_P^{-2}$

- Gauge fixing of *R*-symmetry now leads to

$$0 = \delta Y - \delta \bar{Y} = -\frac{2}{3} \mathrm{i} \Lambda_{\mathrm{U}(1)} Y + (r_\alpha - \bar{r}_\alpha)\Lambda^\alpha$$

Thus a gauge symmetry with non-zero r_α also acts as an *R*-symmetry in supergravity

The D-term and the superpotential

- The D-term (value of auxiliary field) is mathematically related to the moment map

$$D^\alpha = (\mathrm{Re}\, f)^{-1\,\alpha\beta}\mathcal{P}_\beta$$

$$\begin{aligned}\delta_G Y &= Y r_\alpha(z)\Lambda^\alpha \\ \delta_G z^i &= k_\alpha^i(z)\Lambda^\alpha\end{aligned}$$

$$\begin{aligned}\mathcal{P}_\alpha &= \tfrac{1}{2}\mathrm{i} k_\alpha^I \partial_I \mathcal{N} - k_\alpha^{\bar I}\partial_{\bar I}\mathcal{N} \\ &= \tfrac{1}{2}\mathrm{i} M_P^2\left(k_\alpha^i \partial_i K - k_\alpha^{\bar\imath}\partial_{\bar\imath} K\right) + g\xi_\alpha\end{aligned}$$

$$r_\alpha = \tfrac{1}{3}\mathrm{i} g \xi_\alpha M_P^{-2}$$

- Superpotential: contains the compensating scalar:

$$\mathcal{W} = Y^3 W(z)$$

should scale under gauge transformations:

$$3 r_\alpha W + k_\alpha^i \partial_i W = 0$$

K. Stelle and P. West, 1978;
Barbieri, Ferrara, Nanopoulos, Stelle. 1982

basic equation for some recent constraints of susy breaking scenarios
Choi, Falkowski, Nilles, Olechowski 0503216; Villadoro, Zwirner 0508167

Summary on F-term versus D-term

$$\delta_G Y = Y r_\alpha(z)\Lambda^\alpha$$

$$Y\bar{Y}\mathrm{e}^{-K(z,\bar{z})/(3M_P^2)} = M_P^2$$

$$\delta_G K = 3M_P^2\Lambda^\alpha\left(r_\alpha(z) + \bar{r}_\alpha(\bar{z})\right)$$

FI term is $r_\alpha = \frac{1}{3}\mathrm{i}g\xi_\alpha M_P^{-2}$

$$3r_\alpha W + k_\alpha^i \partial_i W = 0$$

- Either W is invariant and $r_\alpha = 0$,
- or W =0 and r_α arbitrary
- or W scales under gauge transformations and weight determines r_α

r_α determines the amount in which G works as R-symmetry

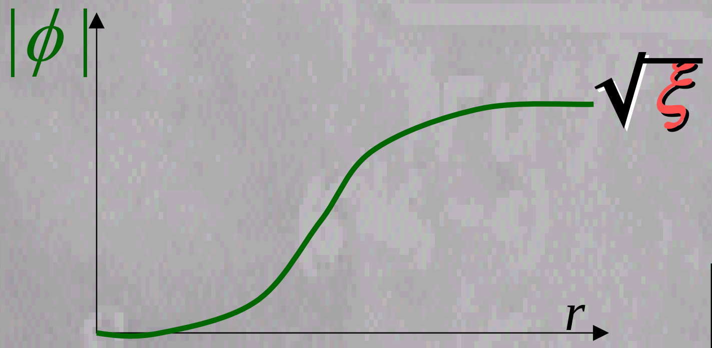
2. Cosmic strings

- the cosmic string solution
- effective supergravity action
- setup from D-branes
- role of FI term

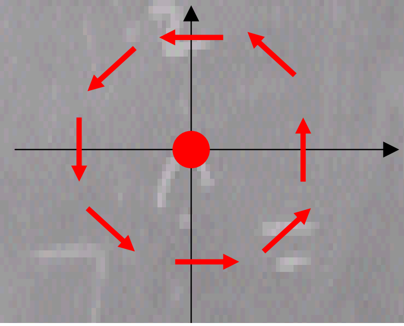
Cosmic string solution

Abrikosov, Nielsen, Olesen

$$V = \frac{1}{2}(D)^2 = \frac{1}{2}g^2\left(\xi - \phi^*\phi\right)^2$$



$D = g\xi$ $D = 0$



$$\begin{aligned} \phi(r,\theta) &= |\phi|(r)\,\mathrm{e}^{\mathrm{i}\theta} \\ gW_\mu\,\mathrm{d}x^\mu &= \alpha(r)\,\mathrm{d}\theta \end{aligned}$$

$$F_{12} = D$$

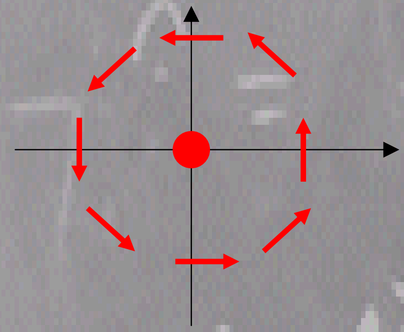
J. Edelstein, C. Núñez and F. Schaposnik, 9506147

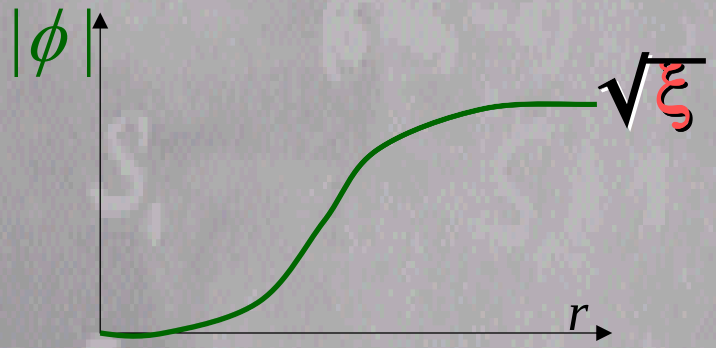
- Supergravity description
  - 1 vector multiplet : gauges U(1) + **FI term**.
  - 1 chiral multiplet with complex scalar: phase transformation under U(1)

Cosmic string solution

1 chiral multiplet (scalar ϕ) charged under *U(1)* of a vector multiplet (W_μ), and a FI term ξ

$$
\begin{aligned}
\phi(r,\theta) &= |\phi|(r)\,\mathrm{e}^{\mathrm{i}\theta} \\
gW_\mu\,\mathrm{d}x^\mu &= \alpha(r)\,\mathrm{d}\theta \\
\mathrm{d}s^2 &= -\mathrm{d}t^2+\mathrm{d}z^2+\mathrm{d}r^2+C^2(r)\mathrm{d}\theta^2\,,
\end{aligned}
$$





$$\delta\lambda = \frac{1}{4}\gamma^{\mu\nu}F_{\mu\nu}\epsilon + \frac{1}{2}\mathrm{i}\gamma_5 D\epsilon$$

$$F_{12} = D = g\xi - g\phi^*\phi$$

BPS solution: ½ susy

$D = g\xi$ $\qquad$ $D = 0$

$C = r$ $\qquad$ $C = r(\,1 - \xi\, M_P^{-2})$

The cosmic string model from branes

G. Dvali, R. Kallosh and AVP, hep-th/0312005

- A supergravity model for the final state after the D3 – $\overline{\text{D3}}$ brane annihilation: a D1 string
- '**FI term**' represents brane-antibrane energy.
- Other correspondences:
  - annihilation is tachyon condensation
  - energy of D_1 brane ↔ energy of string solution
  - tachyon ↔ field ϕ
  - Ramond-Ramond charges from $\int_{3+1} F_{(2)} \wedge C_2$

Magnetic flux in 4 dim.

- Various checks

$$\xi = \frac{1}{4\pi^2 g_s \alpha'} \qquad \text{and} \qquad g^2 = 8\pi g_s$$

Recapitulation

- The cosmic string gets energy from FI term
- Good mechanism, but in cosmology we also need superpotentials to stabilize all the moduli.
- FI term + superpotential ??
  Then superpotential should scale under gauge symmetry
- FI term can be seen as effective result of *D*-terms produced from contributions to *D*-term of multiplets not included in effective action.

P. Binétruy, G. Dvali, R. Kallosh and AVP, 'FI terms in supergravity and cosmology', hep-th/0402046
AVP, Supergravity with Fayet-Iliopoulos terms and R-symmetry, hep-th/0410053.

remark on effective theories

- the FI constants remain also when effective theories are constructed by integrating out multiplets that contributed to the moment map:
  $P = f(\rho) \quad \Longrightarrow \quad P = f(\rho_0)$
- In effective descriptions of string theory one sometimes considers only some chiral multiplets. Others are ‘integrated out’.
  The full string theory may not have explicit FI constants, but the effective theory has the vev of the moment map of these chiral multiplets as FI term

3. *D*-term inflation:
a consistency problem

- Based on a FI term + superpotential

Old D-term inflation model

Binétruy and Dvali, Halyo, 1996

- simplest model: 1 vector multiplet
  and 3 chiral ones $\phi_0 , \phi_\pm$
  with charges $Q_0=0$, $Q_\pm = \pm 1$ and a FI term ξ
- superpotential $W = \lambda \phi_0 \phi_+ \phi_-$

- INCONSISTENT ! If FI term, then superpotential should transform proportional to ξ
- can be remedied by shifting charges with terms proportional to ξM_P^{-2}
- then anomalies require to add 3 more multiplets
- can be remedied at the cost of some more extra terms.

4. D3/D7 KKLT-like model

- KKLT : a strategy to construct a cosmological model with stabilized moduli and **de Sitter vacuum**

S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, hep-th/0301240

- First they construct AdS susy vacuum and then **'the uplift'**. They use D3-$\overline{\text{D3}}$
- Alternative: use D3-D7: a supersymmetric setup: can be described in supergravity
- Fluxes on D7 lead to *D*-terms
- Consistent with supergravity ?

C.P. Burgess, R. Kallosh and F. Quevedo, hep-th/0309187

M. Haack, D. Krefl, D. Lüst, AVP and M. Zagermann, 0609211

Ingredients

- IIB on CY orientifold with D3 and D7 and fluxes.
  D3 and D7 spacetime filling.
  D7 further wrapped over 4-cycles of CY
  $\rightarrow$ N=1 in 4 dimensions
- Moduli T related to the volume of 4-cycle on which D7 is wrapped.

$$K = -2 \ln \frac{1}{6} \int_{CY} J \wedge J \wedge J + \ldots$$

After identifying complex fields of chiral multiplets

$$K = -\ln(T + \bar{T}) + \ldots$$

T is also $f_{\alpha\beta}$ in kinetic U(1) gauge field terms

see talk
J. Louis

some simplifications, e.g. on vanishing odd cohomology of orientifold projection, relative position of branes, …

See more on this setup: T. Grimm, J. Louis, 0403067; T. Grimm, 0507153; D. Lüst, S. Reffert, E. Scheidegger, W. Schulgin and S. Stieberger, 0609013

D-terms with Kähler moduli

- We saw before that the *D*-term is given by

$$V_D = \frac{1}{2}\mathcal{P}_\alpha \mathcal{P}_\beta (\mathrm{Re}\, f)^{-1\,\alpha\beta} \simeq \frac{1}{2} g^2 \mathcal{P}^2$$

$$\mathcal{P}_\alpha = \frac{1}{2}\mathrm{i} M_P^2 \left(k_\alpha^i \partial_i K - k_\alpha^{\bar{\imath}} \partial_{\bar{\imath}} K \right) + g\xi_\alpha$$

- For Kähler potentials like $K = -\ln(T+\bar{T}) + \bar{\phi}^I \phi^I$

  ∃ isometries $\delta T = \mathrm{i} q \Lambda, \qquad \delta\phi^I = \mathrm{i} q_I \phi^I$

when these couple to a vector multiplet it generates

$$\mathcal{P} = M_P^2 \left(-\frac{q}{T+\bar{T}} + q_I \bar{\phi}^I \phi^I \right)$$

with a stabilized modulus *T* this is similar to a FI term and generates an uplifting

Can ϕ^I be zero or can their vevs be such that $D \neq 0$?

Burgess, Kallosh, Quevedo; Binétruy, Dudas; Achúcarro, de Carlos, Casas, Doplicher

Answer is model-dependent.

Problem superpotential

$$W = W_{\mathsf{flux}} + W_{\mathsf{np}}$$

→ related to gaugino condensation, or Euclidean D3 instanton

recent similar results on Eucl.D3: R. Blumenhagen, M. Cvetič and T. Weigand, 0609091

$$W_{\mathsf{np}} = A(z)\mathrm{e}^{-\alpha T^{\mathbb{G}}}$$

z: open string fields, complex structure moduli, dilaton

$T^{\mathbb{G}}$: Kähler modulus measuring the volume of 4-cycle on which D7 for gaugino condensation are wrapped

- *D*-term should come from charge of these moduli under gauge group related to branes with flux.
- $\exists$ other terms in $W \rightarrow$ FI-term excluded (superpotential should scale homogeneously). Therefore W_{np} should be invariant.
- How is this realized ?

Brane setup

- $D7_{\mathbb{G}}$ brane stack for gaugino condensation
  gauge group $U(N_{\mathbb{G}})$
  wrap 4-cycles $\Sigma^{\mathbb{G}}$ with modulus $T^{\mathbb{G}}$
- $D7_{\mathbb{F}}$ brane stack with fluxes
  gauge group $U(1)_{\mathbb{F}} \times SU(N_{\mathbb{F}})$
  wrap 4-cycles $\Sigma^{\mathbb{F}}$ with modulus $T^{\mathbb{F}}$
- Open string fields Φ_{ia} (quarks and anti-quarks)
  stretching between $D7_{\mathbb{G}}$ and $D7_{\mathbb{F}}$ **charged under $U(1)_{\mathbb{F}}$**
- Non-trivial flux on 2-cycle $\Sigma^{\mathbb{F}} \cap \Sigma^{\mathbb{G}}$
  determines shift symmetry of $T^{\mathbb{G}}$ under $U(1)_{\mathbb{F}}$

This setup also in H. Jockers and J. Louis, 0502059

Cancellation of charges

- After gaugino condensation: Affleck-Dine-Seiberg superpotential (or Taylor-Veneziano-Yankielowicz)

$$W_{\mathrm{ADS}} = \left(\frac{\mathrm{e}^{-8\pi^2 T^{\mathbb{G}}}}{\det M}\right)^{\frac{1}{N_{\mathbb{G}}-N_F}}$$

Dudas, Vempati, 0506029;
Achúcarro, de Carlos,
Casas, Doplicher, 0601190

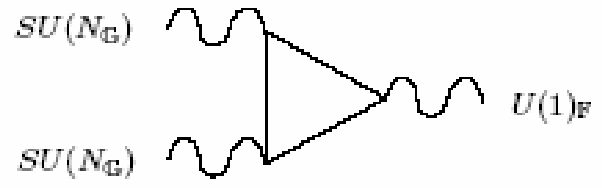
$$\det M \equiv \det(\tilde{\Phi}^{ia}\Phi_{ja}),$$

see talk A. Bilal

U(1) charges should cancel

- Similar non-invariance before gaugino condensation related to anomalies

$$\int (\mathrm{Im}\, T^{\mathbb{G}})\, \mathrm{tr}\, F^{\mathbb{G}} \wedge F^{\mathbb{G}}$$



we need number of bifundamentals and q in $\delta T = \mathrm{i} q \Lambda$

From string theory

- Net number of $U(1)_{\mathbb{F}}$ charges of quarks-antiquarks given by index of Dirac operator at intersection in flux background

$$\text{index}(\nabla) = \alpha'^{-1} \int_{\Sigma^{\mathbb{F}} \cap \Sigma^{\mathbb{G}}} \hat{A}(T(\Sigma^{\mathbb{F}} \cap \Sigma^{\mathbb{G}})) \wedge \text{ch}\, F = \alpha'^{-1} \int_{\Sigma^{\mathbb{F}} \cap \Sigma^{\mathbb{G}}} \frac{F}{2\pi} = Q_{\mathbb{GF}}$$

see talk L. Alvarez-Gaumé

further factor $2N_{\mathbb{F}}$

- Charge q of $T^{\mathbb{G}}$ either from
  - expansion of DBI term on $D7_{\mathbb{F}}$ to give *D*-term
  - or from WZ term

$$q = \frac{1}{4\pi^2} N_{\mathbb{F}} Q_{\mathbb{GF}}$$

$$W_{\text{ADS}} = \left(\frac{e^{-8\pi^2 T^{\mathbb{G}}}}{\det M} \right)^{\frac{1}{N_{\mathbb{G}} - N_F}}$$ invariant, and D-term generated !

generation of D-term with other methods: H. Jockers and J. Louis, 0502059; D-term generation also in G. Villadoro and F. Zwirner, 0508167

M. Haack, D. Krefl, D. Lüst, AVP and M. Zagermann, 0609211

We obtained a further step in generating a cosmological model

- Following the strategy of KKLT, but using D7 branes rather than $\overline{\text{D3}}$, we could obtain an embedding in supergravity starting from superstrings elements and the brane actions.
- In the paper, we also considered the higher curvature corrections and these do not spoil the picture.
- The model is not yet complete. With this content: SU($N_{\mathbb{F}}$) would still be anomalous. Needs more U(1), generalized Chern-Simons terms, …

B.de Wit, P.Lauwers, AVP, 1985; L. Andrianopoli, S. Ferrara, M. Lledó, 0402142; P. Anastasopoulos, M. Bianchi, E. Dudas and E. Kiritsis, 0605225

5. *N*=2 *D*-terms

- What are the D-terms and FI terms in *N*=2 ?
- Difference supersymmetry and supergravity.
- Cosmic strings in *N*=2.

$\mathcal{N}=1$ and $\mathcal{N}=2$ supergravities

$\mathcal{N}=1$			$\mathcal{N}=2$	
graviton m.	$(2,\frac{3}{2})$		graviton m.	$(2,\frac{3}{2},\frac{3}{2},1)$
vector mult.	$(1,\frac{1}{2})$		vector mult.	$(1,\frac{1}{2},\frac{1}{2},0,0)$ (special) Kähler geometry
chiral mult.	$(\frac{1}{2},0,0)$ Kähler geometry		hypermult.	$(\frac{1}{2},\frac{1}{2},0,0,0,0)$ quaternionic-Kähler

D-term (FI term): aux. fields of vector multiplets determined by isometries of chiral multiplets

moment map $\mathcal{P}_\alpha$

D-term (FI term): aux. fields of vector multiplets determined by isometries of hypermultiplets

triplet moment map $\vec{\mathcal{P}}_\alpha$

Moment map and FI terms

RIGID SUSY

(triplet) moment maps determined from isometries

$$\mathrm{d}\mathcal{P} = \iota_k J$$

Killing vectors on Kähler manifold

$$\mathrm{d}\vec{\mathcal{P}} = \iota_k \vec{J}$$

Killing vectors on hyper-Kähler manifold

FI terms are undetermined constants in $\mathcal{P}$

SUGRA

Use cone structure, i.e. impose conformal symmetry.
This does not allow anymore to add constants

FI terms are transformations of compensating fields

Cosmic strings from N=2

- Triplet moment map gives uplifting terms

Ana Achúcarro, Alessio Celi, Mboyo Esole, Joris Van den Bergh and AVP, 0511001

- Fields that are effective for string can be considered as a consistent truncation of N=2 to N=1

$$\begin{array}{ccc} \mathcal{L}_{N=2} & \xrightarrow{\frac{\delta}{\delta\Phi}} & \text{e.o.m} \\ \text{ansatz}\downarrow & & \downarrow\text{ansatz} \\ \mathcal{L}_{N=1} & \xrightarrow{\frac{\delta}{\delta\Phi}} & \text{e.o.m} \end{array}$$

see talk C.Pope

leads to effective FI term in N=1

N=2 consistent truncations have been considered in details in papers of L. Andrianopoli, R. D'Auria and S. Ferrara, 2001

6. Final remarks

- I learned all these techniques from the supergravity community.
- Supergravity was a beautiful subject for the last 27 years for me. Thanks to all of you who taught me the subject and to all my collaborators.
- Often I thought that we reached the end of the possibilities of supergravity calculations, but this talk showed that there are more applications.
- Supergravity is more alive than ever !
  I do still recommend students to start in this field.