MAXIMAL SUPERSYMMETRY

"30 years of supergravity"

ENS, Paris, 16-20 october

H. Nicolai

(AEI, Golm)

Symmetry = (arguably) most successful guiding principle of physics

$\Rightarrow$ search for maximally symmetric theories still best strategy for unification of fundamental interactions & consistent quantization of gravity

Supersymmetry (SUSY) = enlargement of conventional (= Lie algebra) notion of symmetry $\longrightarrow$ B. Zumino
J. Wess

Local SUSY $\longleftrightarrow$ Supergravity (SUGRA)

$\longrightarrow$ P. van Nieuwenhuizen

SUSY algebras exist for all

N = # (supercharges)

D = # (dimensions of space-time)

However, consistent and interacting SUSY field theories appear to exist only for limited values

$$\boxed{N \leq 32 \iff D \leq 11}$$

[for global SUSY : $N \leq 16 \Leftrightarrow D \leq 10$]

Search for maximally supersymmetric theories has led to **maximal point FT's**

* $D = 11$ SUGRA (CJS, 1978)
* $D = 4, N = 8$ SUGRA (CJ, 1979; dWN, 1982)

as well as maximally supersymmetric **extended object theories:**

* Type II superstrings (GS, 1982) → M. Green, J. Schwarz
* $D = 11$ supermembrane (BST, 1987)

$\updownarrow$ (dWHN, 1988)

M(atrix) Theory $\equiv \mathcal{N} \to \infty$ SUSY YM

which contain $D = 10$ & $D = 11$ SUGRAs as low energy limits (in some sense)

Remarkable fact: there are distinguished theories in the "space of all theories"!

Drawback: rigid structure of these theories makes it difficult to establish link with the "real world" ...

Bounds on N and D may be related to:

- absence of massless spin > 2

  (Nahm, 1976)

- connection with maximally extended Kac-Moody-Lie algebras E_8 and E_{10}

  $\longrightarrow$ B. Julia, P. West

<u>Important Caveat</u>: all constructions based on "conventional" notions of space-time, but quantum gravity very likely requires more than a space-time based QFT $\longrightarrow$

- non-commutative space-time, quantum geometry & symmetry?
- emergent space-time?

  [e.g. as suggested by link between BKL cosmology and E_{10} (DHN)]
- other ideas?

$\rightarrow$ may be relevant also for question of SUSY breaking!

D = 11 : the mother of all SUGRAs?

(CJS, 1978)

Field content: $G_{MN}\,(44)$, $A_{MNP}\,(84)$, $\Psi_M\,(128)$

Lagrangian:

$$E^{-1}\mathcal{L} = \frac{1}{4}R - \frac{i}{2}\bar{\Psi}_M \Gamma^{MNP} D_N \Psi_P - \frac{1}{48}F^2_{MNPQ}$$

$$- \frac{i}{96}\left(\bar{\Psi}_M \Gamma^{MNPQRS}\Psi_N + 12\,\bar{\Psi}^P \Gamma^{QR}\Psi^S\right) F_{PQRS}$$

$$+ \frac{2}{(12)^4}\,\varepsilon^{M_1\ldots M_{11}} F_{M_1\ldots M_4} F_{M_5\ldots M_8} A_{M_9 M_{10} M_{11}}$$

$$+ \text{ quartic terms}$$

Unique theory:

* no matter couplings
* not deformable
* maximal space-time dimension compatible with SUSY
* multiplet structure "explained" by $E_{n(n)}$

Furthermore:

* compactifies preferentially via KK to $D = 4$ (and $D = 7$) (FR, 1980)
  $\longrightarrow$ M. Duff
* strong coupling limit of superstring theory? (Witten, 1995)

Kaluza Klein compactification to $\mathcal{D}=4$:

$$\begin{tikzcd}[column sep=small]
 & & \mathcal{D}=11 \text{ SUGRA} \arrow[dll, "T^7"'] \arrow[dl, "S^7"] \arrow[d, "\text{``non-compactification''}"] \arrow[dr, "\text{Scherk-Schwarz \& flux compactificatio}"] & \\
CJ & SO(8) & SO(p,8-p) & \text{non-semisimple gaugings}
\end{tikzcd}$$

→ S. Ferrara

Also contains ungauged maximal SUGRAs $\forall \mathcal{D}$ via dimensional reduction on $T^{11-\mathcal{D}}$.

Idem: $\mathcal{D}=11$ SUGRA on $AdS^7 \times S^4$
IIB SUGRA on $AdS^5 \times S^5$

However, not all maximal theories in $\mathcal{D}<11$ have higher dimensional origin

→ Richest variety of maximal gauged SUGRAs for $\mathcal{D}=3$: (Samtleben, N., 2000)

* deformations of $\mathcal{D}=3$, $N=16$ without $\mathcal{D}>3$ origin
* numerous semisimple and non-semisimple $G_0 \subset E_{8(8)}$
* KK compactifications to $\mathcal{D}=3$ $\leftrightarrow$ <u>non</u>-semisimple G_0, e.g.

IIA on T^7 $\leftrightarrow$ $G_0 = SO(8) \ltimes T_{28}$

Maximal $D=3$ SUGRAs with semisimple G_0

gauge group G_0	ratio g_1/g_2	(n_L, n_R)	ground state symmetry group
$SO(8)\times SO(8)$	$g_1/g_2 = -1$	$(8,8)$	$OSp(8\|2,\mathbb{R})\times OSp(8\|2,\mathbb{R})$
$SO(7,1)\times SO(7,1)$	$g_1/g_2 = -1$	$(8,8)$	$F(4)\times F(4)$
$SO(6,2)\times SO(6,2)$	$g_1/g_2 = -1$	$(8,8)$	$SU(4\|1,1)\times SU(4\|1,1)$
$SO(5,3)\times SO(5,3)$	$g_1/g_2 = -1$	$(8,8)$	$OSp(4^*\|4)\times OSp(4^*\|4)$
$SO(4,4)\times SO(4,4)$	$g_1/g_2 = -1$	$(8,8)$	Minkowski vacuum
$G_{2(2)}\times F_{4(4)}$	$g_{G_2}/g_{F_4} = -3/2$	$(4,12)$	$D^1(2,1;-\frac{2}{3})\times OSp(4^*\|6)$
$G_2\times F_{4(-20)}$	$g_{G_2}/g_{F_4} = -3/2$	$(7,9)$	$G(3)\times OSp(9\|2,\mathbb{R})$
$E_{6(6)}\times SL(3)$	$g_{A_2}/g_{E_6} = -2$	$(16,0)$	$OSp(4^*\|8)\times SU(1,1)$
$E_{6(2)}\times SU(2,1)$	$g_{A_2}/g_{E_6} = -2$	$(12,4)$	$SU(6\|1,1)\times D^1(2,1;-\frac{1}{2})$
$E_{6(-14)}\times SU(3)$	$g_{A_2}/g_{E_6} = -2$	$(10,6)$	$OSp(10\|2,\mathbb{R})\times SU(3\|1,1)$
$E_{7(7)}\times SL(2)$	$g_{A_1}/g_{E_7} = -3$	$(16,0)$	$SU(8\|1,1)\times SU(1,1)$
$E_{7(-5)}\times SU(2)$	$g_{A_1}/g_{E_7} = -3$	$(12,4)$	$OSp(12\|2,\mathbb{R})\times D^1(2,1;-\frac{1}{3})$
$E_{8(8)}$	g_{E_8}	$(16,0)$	$OSp(16\|2,\mathbb{R})\times SU(1,1)$

Table 2: The $N=16$ theories with semisimple gauge groups G_0. Except for the last row, the gauge groups appear as direct products of two factors whose coupling constant ratio g_1/g_2 is determined by (9.2). All these theories admit a maximally supersymmetric AdS (or Minkowski, for $G_0 = SO(4,4)\times SO(4,4)$) ground state, whose symmetry group factorizes according to $G_L\times G_R$, as specified in the last column; the supercharges split accordingly into $n_L + n_R = 16$.

as well as $G_0 = SO(8,\mathbb{C}) \subset E_{8(8)}$:
no isometries, unstable de Sitter!

Further peculiarities of $D=3$:

* Fully SUSY (hence, stable) vacua also for non-compact gaugings

* $\exists$ stable AdS ground states with fully broken SUSY

Summary: fairly complete picture of bestiary of maximal SUSY theories, but several "old" problems persist:

* Off-shell formulation (components or superspace) of maximal theories?

* Finiteness (or not) of $N = 8$ SUGRA?
  [NB: $D = 11$ SUGRA <u>is</u> divergent!]

  → P. Townsend, S. Deser

* Maximal SUSY and higher order corrections (R^2, R^3, ...)?

* $E_{n(n)}$ and higher order corrections?
  (e.g. $E_{n(n)} \longrightarrow E_n(\mathbb{Z})$?)

and, perhaps, less importantly:

* Extremal structure of gauged SUGRA potentials:
  - $N = 8$, $D = 4$: no progress since Warner ('8
  - $N = 16$, $D = 3$: much more complicated
    e.g. $G_0 = G_2 \times F_{4(-20)}$ on subspace of $SU(3) \times SU(3)$ singlets

(T. Fischbacher, PhD Thesis)

$-\frac{1}{32}\cosh(s)\cosh(z)\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)$
$+\frac{1}{128}\cosh(s)\cosh(2z)\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)$
$+\frac{1}{128}\cosh(2s)\cosh(z)\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)$
$-\frac{1}{512}\cosh(2s)\cosh(2z)\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)$
$+\frac{7}{8}\cosh(s)\cosh(z)\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)$
$-\frac{1}{128}\cosh(s)\cosh(z)\cos(8r_1-8r_4)\cos(8r_2)\cos(8r_5)$
$+\frac{1}{32}\cosh(s)\cosh(2z)\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)$
$+\frac{1}{512}\cosh(s)\cosh(2z)\cos(8r_1-8r_4)\cos(8r_2)\cos(8r_5)$
$+\frac{1}{32}\cosh(2s)\cosh(z)\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)$
$+\frac{1}{512}\cosh(2s)\cosh(z)\cos(8r_1-8r_4)\cos(8r_2)\cos(8r_5)$
$-\frac{5}{128}\cosh(2s)\cosh(2z)\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)$
$-\frac{1}{2048}\cosh(2s)\cosh(2z)\cos(8r_1-8r_4)\cos(8r_2)\cos(8r_5)$
$+\frac{1}{64}\cosh(2s)\cosh(2z)\sin(4r_3-4r_6)\sin(4r_2)\sin(4r_1-4r_4)$
$+\frac{1}{64}\sin(2r_3-2r_6)\sin(2r_2-6r_5)\sin(2r_1-2r_4)\sinh(z)\sinh(s)$
$+\frac{229}{64}\sin(2r_3-2r_6)\sin(2r_2+2r_5)\sin(2r_1-2r_4)\sinh(z)\sinh(s)$
$-\frac{1}{64}\sin(2r_3-2r_6)\sin(6r_2-2r_5)\sin(2r_1-2r_4)\sinh(z)\sinh(s)$
$+\frac{3}{64}\sin(2r_3-2r_6)\sin(6r_2+6r_5)\sin(2r_1-2r_4)\sinh(z)\sinh(s)$
$+\frac{1}{64}\sin(2r_3-2r_6)\sin(2r_2-6r_5)\sin(6r_1-6r_4)\sinh(z)\sinh(s)$
$+\frac{1}{64}\sin(2r_3-2r_6)\sin(2r_2+2r_5)\sin(6r_1-6r_4)\sinh(z)\sinh(s)$
$-\frac{1}{64}\sin(2r_3-2r_6)\sin(6r_2-2r_5)\sin(6r_1-6r_4)\sinh(z)\sinh(s)$
$-\frac{1}{64}\sin(2r_3-2r_6)\sin(6r_2+6r_5)\sin(6r_1-6r_4)\sinh(z)\sinh(s)$
$-\frac{1}{128}\sin(2r_3-2r_6)\sin(2r_2-6r_5)\sin(2r_1-2r_4)\sinh(2z)\sinh(s)$
$-\frac{5}{128}\sin(2r_3-2r_6)\sin(2r_2+2r_5)\sin(2r_1-2r_4)\sinh(2z)\sinh(s)$
$+\frac{1}{128}\sin(2r_3-2r_6)\sin(6r_2-2r_5)\sin(2r_1-2r_4)\sinh(2z)\sinh(s)$
$-\frac{3}{128}\sin(2r_3-2r_6)\sin(6r_2+6r_5)\sin(2r_1-2r_4)\sinh(2z)\sinh(s)$
$-\frac{1}{128}\sin(2r_3-2r_6)\sin(2r_2-6r_5)\sin(6r_1-6r_4)\sinh(2z)\sinh(s)$
$-\frac{1}{128}\sin(2r_3-2r_6)\sin(2r_2+2r_5)\sin(6r_1-6r_4)\sinh(2z)\sinh(s)$
$+\frac{1}{128}\sin(2r_3-2r_6)\sin(6r_2-2r_5)\sin(6r_1-6r_4)\sinh(2z)\sinh(s)$
$+\frac{1}{128}\sin(2r_3-2r_6)\sin(6r_2+6r_5)\sin(6r_1-6r_4)\sinh(2z)\sinh(s)$
$-\frac{1}{128}\sin(2r_3-2r_6)\sin(2r_2-6r_5)\sin(2r_1-2r_4)\sinh(z)\sinh(2s)$
$-\frac{5}{128}\sin(2r_3-2r_6)\sin(2r_2+2r_5)\sin(2r_1-2r_4)\sinh(z)\sinh(2s)$
$+\frac{1}{128}\sin(2r_3-2r_6)\sin(6r_2-2r_5)\sin(2r_1-2r_4)\sinh(z)\sinh(2s)$
$-\frac{3}{128}\sin(2r_3-2r_6)\sin(6r_2+6r_5)\sin(2r_1-2r_4)\sinh(z)\sinh(2s)$
$-\frac{1}{128}\sin(2r_3-2r_6)\sin(2r_2-6r_5)\sin(6r_1-6r_4)\sinh(z)\sinh(2s)$
$-\frac{1}{128}\sin(2r_3-2r_6)\sin(2r_2+2r_5)\sin(6r_1-6r_4)\sinh(z)\sinh(2s)$
$+\frac{1}{128}\sin(2r_3-2r_6)\sin(6r_2-2r_5)\sin(6r_1-6r_4)\sinh(z)\sinh(2s)$
$+\frac{1}{128}\sin(2r_3-2r_6)\sin(6r_2+6r_5)\sin(6r_1-6r_4)\sinh(z)\sinh(2s)$
$+\frac{1}{256}\sin(2r_3-2r_6)\sin(2r_2-6r_5)\sin(2r_1-2r_4)\sinh(2z)\sinh(2s)$
$-\frac{27}{256}\sin(2r_3-2r_6)\sin(2r_2+2r_5)\sin(2r_1-2r_4)\sinh(2z)\sinh(2s)$
$-\frac{1}{256}\sin(2r_3-2r_6)\sin(6r_2-2r_5)\sin(2r_1-2r_4)\sinh(2z)\sinh(2s)$
$+\frac{3}{256}\sin(2r_3-2r_6)\sin(6r_2+6r_5)\sin(2r_1-2r_4)\sinh(2z)\sinh(2s)$
$+\frac{1}{256}\sin(2r_3-2r_6)\sin(2r_2-6r_5)\sin(6r_1-6r_4)\sinh(2z)\sinh(2s)$

$+\frac{1}{64}\cosh(2s)\cos(4r_1-4r_4)\cos(4r_3-4r_6)$
$-\frac{1}{64}\cosh(2s)\sin(4r_5)\sin(4r_3-4r_6)\sin(4r_1-4r_4)$
$+\frac{3}{512}\cosh(s)\cos(8r_1-8r_4)\cos(8r_2)$
$-\frac{3}{2048}\cosh(2s)\cos(8r_1-8r_4)\cos(8r_2)$
$+\frac{3}{128}\cosh(s)\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)$
$-\frac{3}{512}\cosh(2s)\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)$
$-\frac{29}{32}\cosh(s)\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)$
$+\frac{3}{512}\cosh(s)\cos(8r_1-8r_4)\cos(8r_2)\cos(8r_5)$
$+\frac{1}{128}\cosh(2s)\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)$
$-\frac{3}{2048}\cosh(2s)\cos(8r_1-8r_4)\cos(8r_2)\cos(8r_5)$
$-\frac{1}{64}\cosh(2s)\sin(4r_3-4r_6)\sin(4r_2)\sin(4r_1-4r_4)$
$+\frac{1}{64}\cosh(2s)\cos(4r_1-4r_4)\cos(4r_3-4r_6)\sin(4r_5)\sin(4r_2)$
$+\frac{241}{128}\cosh(s)\cosh(z)-\frac{21}{512}\cosh(s)\cosh(2z)$
$-\frac{21}{512}\cosh(2s)\cosh(z)-\frac{107}{2048}\cosh(2s)\cosh(2z)$
$+\frac{1}{128}\cosh(s)\cosh(z)\cos(8r_5)-\frac{1}{512}\cosh(s)\cosh(2z)\cos(8r_5)$
$-\frac{5}{512}\cosh(2s)\cosh(z)\cos(8r_5)+\frac{1}{2048}\cosh(2s)\cosh(2z)\cos(8r_5)$
$+\frac{1}{128}\cosh(s)\cosh(z)\cos(8r_2)-\frac{1}{512}\cosh(s)\cosh(2z)\cos(8r_2)$
$-\frac{1}{512}\cosh(2s)\cosh(z)\cos(8r_2)+\frac{1}{2048}\cosh(2s)\cosh(2z)\cos(8r_2)$
$+\frac{7}{8}\cosh(s)\cosh(z)\sin(4r_5)\sin(4r_2)$
$+\frac{1}{32}\cosh(s)\cosh(2z)\sin(4r_5)\sin(4r_2)$
$+\frac{1}{32}\cosh(2s)\cosh(z)\sin(4r_5)\sin(4r_2)$
$-\frac{5}{128}\cosh(2s)\cosh(2z)\sin(4r_5)\sin(4r_2)$
$-\frac{3}{128}\cosh(s)\cosh(z)\cos(8r_2)\cos(8r_5)$
$+\frac{3}{512}\cosh(s)\cosh(2z)\cos(8r_2)\cos(8r_5)$
$+\frac{3}{512}\cosh(2s)\cosh(z)\cos(8r_2)\cos(8r_5)$
$-\frac{3}{2048}\cosh(2s)\cosh(2z)\cos(8r_2)\cos(8r_5)$
$-\frac{1}{64}\cosh(2s)\cosh(2z)\cos(4r_2)\cos(4r_3-4r_6)\cos(4r_5)$
$-\frac{1}{128}\cosh(s)\cosh(z)\cos(8r_1-8r_4)$
$+\frac{1}{512}\cosh(s)\cosh(2z)\cos(8r_1-8r_4)$
$+\frac{1}{512}\cosh(2s)\cosh(z)\cos(8r_1-8r_4)$
$-\frac{1}{2048}\cosh(2s)\cosh(2z)\cos(8r_1-8r_4)$
$-\frac{1}{128}\cosh(s)\cosh(z)\cos(8r_1-8r_4)\cos(8r_5)$
$+\frac{1}{512}\cosh(s)\cosh(2z)\cos(8r_1-8r_4)\cos(8r_5)$
$+\frac{1}{512}\cosh(2s)\cosh(z)\cos(8r_1-8r_4)\cos(8r_5)$
$-\frac{1}{2048}\cosh(2s)\cosh(2z)\cos(8r_1-8r_4)\cos(8r_5)$
$-\frac{1}{64}\cosh(2s)\cosh(2z)\cos(4r_1-4r_4)\cos(4r_3-4r_6)$
$+\frac{1}{64}\cosh(2s)\cosh(2z)\sin(4r_5)\sin(4r_3-4r_6)\sin(4r_1-4r_4)$
$-\frac{1}{128}\cosh(s)\cosh(z)\cos(8r_1-8r_4)\cos(8r_2)$
$+\frac{1}{512}\cosh(s)\cosh(2z)\cos(8r_1-8r_4)\cos(8r_2)$
$+\frac{1}{512}\cosh(2s)\cosh(z)\cos(8r_1-8r_4)\cos(8r_2)$
$-\frac{1}{2048}\cosh(2s)\cosh(2z)\cos(8r_1-8r_4)\cos(8r_2)$

$$= \frac{24125}{2048} + \frac{9}{2048}\cos(8r_5) + \frac{9}{2048}\cos(8r_2)$$
$$+\frac{115}{128}\sin(4r_5)\sin(4r_2) - \frac{27}{2048}\cos(8r_2)\cos(8r_5)$$
$$-\frac{1}{64}\cos(4r_2)\cos(4r_3-4r_6)\cos(4r_5) - \frac{9}{2048}\cos(8r_1-8r_4)$$
$$-\frac{9}{2048}\cos(8r_1-8r_4)\cos(8r_5) - \frac{1}{64}\cos(4r_1-4r_4)\cos(4r_3-4r_6)$$
$$+\frac{1}{64}\sin(4r_5)\sin(4r_3-4r_6)\sin(4r_1-4r_4) - \frac{9}{2048}\cos(8r_1-8r_4)\cos(8r_2)$$
$$-\frac{9}{512}\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)$$
$$+\frac{115}{128}\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)$$
$$-\frac{9}{2048}\cos(8r_1-8r_4)\cos(8r_2)\cos(8r_5)$$
$$+\frac{1}{64}\sin(4r_3-4r_6)\sin(4r_2)\sin(4r_1-4r_4)$$
$$-\frac{1}{64}\cos(4r_1-4r_4)\cos(4r_3-4r_6)\sin(4r_5)\sin(4r_2) + \frac{449}{512}\cosh(z)$$
$$-\frac{65}{2048}\cosh(2z) - \frac{3}{512}\cosh(z)\cos(8r_5) + \frac{3}{2048}\cosh(2z)\cos(8r_5)$$
$$-\frac{3}{512}\cosh(z)\cos(8r_2) + \frac{3}{2048}\cosh(2z)\cos(8r_2)$$
$$-\frac{29}{32}\cosh(z)\sin(4r_5)\sin(4r_2) + \frac{1}{128}\cosh(2z)\sin(4r_5)\sin(4r_2)$$
$$+\frac{9}{512}\cosh(z)\cos(8r_2)\cos(8r_5) - \frac{9}{2048}\cosh(2z)\cos(8r_2)\cos(8r_5)$$
$$+\frac{1}{64}\cosh(2z)\cos(4r_2)\cos(4r_3-4r_6)\cos(4r_5) + \frac{3}{512}\cosh(z)\cos(8r_1-8r_4)$$
$$-\frac{3}{2048}\cosh(2z)\cos(8r_1-8r_4) + \frac{3}{512}\cosh(z)\cos(8r_1-8r_4)\cos(8r_5)$$
$$-\frac{3}{2048}\cosh(2z)\cos(8r_1-8r_4)\cos(8r_5)$$
$$+\frac{1}{64}\cosh(2z)\cos(4r_1-4r_4)\cos(4r_3-4r_6)$$
$$-\frac{1}{64}\cosh(2z)\sin(4r_5)\sin(4r_3-4r_6)\sin(4r_1-4r_4)$$
$$+\frac{3}{512}\cosh(z)\cos(8r_1-8r_4)\cos(8r_2)$$
$$-\frac{3}{2048}\cosh(2z)\cos(8r_1-8r_4)\cos(8r_2)$$
$$+\frac{3}{128}\cosh(z)\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)$$
$$-\frac{3}{512}\cosh(2z)\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)$$
$$-\frac{29}{32}\cosh(z)\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)$$
$$+\frac{3}{512}\cosh(z)\cos(8r_1-8r_4)\cos(8r_2)\cos(8r_5)$$
$$+\frac{1}{128}\cosh(2z)\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)$$
$$-\frac{3}{2048}\cosh(2z)\cos(8r_1-8r_4)\cos(8r_2)\cos(8r_5)$$
$$-\frac{1}{64}\cosh(2z)\sin(4r_3-4r_6)\sin(4r_2)\sin(4r_1-4r_4)$$
$$+\frac{1}{64}\cosh(2z)\cos(4r_1-4r_4)\cos(4r_3-4r_6)\sin(4r_5)\sin(4r_2)$$
$$+\frac{449}{512}\cosh(s) - \frac{65}{2048}\cosh(2s) - \frac{3}{512}\cosh(s)\cos(8r_5)$$
$$+\frac{3}{2048}\cosh(2s)\cos(8r_5) - \frac{3}{512}\cosh(s)\cos(8r_2)$$
$$+\frac{3}{2048}\cosh(2s)\cos(8r_2) - \frac{29}{32}\cosh(s)\sin(4r_5)\sin(4r_2)$$
$$+\frac{1}{128}\cosh(2s)\sin(4r_5)\sin(4r_2) + \frac{9}{512}\cosh(s)\cos(8r_2)\cos(8r_5)$$
$$-\frac{9}{2048}\cosh(2s)\cos(8r_2)\cos(8r_5)$$
$$+\frac{1}{64}\cosh(2s)\cos(4r_2)\cos(4r_3-4r_6)\cos(4r_5)$$
$$+\frac{3}{512}\cosh(s)\cos(8r_1-8r_4) - \frac{3}{2048}\cosh(2s)\cos(8r_1-8r_4)$$
$$+\frac{3}{\ldots}\cosh(s)\cos(8r_1-8r_4)\cos(8r_5)$$

$$+\frac{1}{256}\sin(2r_3-2r_6)\sin(2r_2+2r_5)\sin(6r_1-6r_4)\sinh(2z)\sinh(2s)$$
$$-\frac{1}{256}\sin(2r_3-2r_6)\sin(6r_2-2r_5)\sin(6r_1-6r_4)\sinh(2z)\sinh(2s)$$
$$-\frac{1}{256}\sin(2r_3-2r_6)\sin(6r_2+6r_5)\sin(6r_1-6r_4)\sinh(2z)\sinh(2s)$$
$$-\frac{229}{64}\cos(2r_1-2r_4)\cos(2r_2-2r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(s)$$
$$-\frac{1}{64}\cos(2r_1-2r_4)\cos(2r_2+6r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(s)$$
$$+\frac{3}{64}\cos(2r_1-2r_4)\cos(6r_2-6r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(s)$$
$$-\frac{1}{64}\cos(2r_1-2r_4)\cos(6r_2+2r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(s)$$
$$+\frac{1}{64}\cos(6r_1-6r_4)\cos(2r_2-2r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(s)$$
$$+\frac{1}{64}\cos(6r_1-6r_4)\cos(2r_2+6r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(s)$$
$$+\frac{1}{64}\cos(6r_1-6r_4)\cos(6r_2-6r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(s)$$
$$+\frac{1}{64}\cos(6r_1-6r_4)\cos(6r_2+2r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(s)$$
$$+\frac{5}{128}\cos(2r_1-2r_4)\cos(2r_2-2r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(s)$$
$$+\frac{1}{128}\cos(2r_1-2r_4)\cos(2r_2+6r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(s)$$
$$-\frac{3}{128}\cos(2r_1-2r_4)\cos(6r_2-6r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(s)$$
$$+\frac{1}{128}\cos(2r_1-2r_4)\cos(6r_2+2r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(s)$$
$$-\frac{1}{128}\cos(6r_1-6r_4)\cos(2r_2-2r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(s)$$
$$-\frac{1}{128}\cos(6r_1-6r_4)\cos(2r_2+6r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(s)$$
$$-\frac{1}{128}\cos(6r_1-6r_4)\cos(6r_2-6r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(s)$$
$$-\frac{1}{128}\cos(6r_1-6r_4)\cos(6r_2+2r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(s)$$
$$+\frac{5}{128}\cos(2r_1-2r_4)\cos(2r_2-2r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(2s)$$
$$+\frac{1}{128}\cos(2r_1-2r_4)\cos(2r_2+6r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(2s)$$
$$-\frac{3}{128}\cos(2r_1-2r_4)\cos(6r_2-6r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(2s)$$
$$+\frac{1}{128}\cos(2r_1-2r_4)\cos(6r_2+2r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(2s)$$
$$-\frac{1}{128}\cos(6r_1-6r_4)\cos(2r_2-2r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(2s)$$
$$-\frac{1}{128}\cos(6r_1-6r_4)\cos(2r_2+6r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(2s)$$
$$-\frac{1}{128}\cos(6r_1-6r_4)\cos(6r_2-6r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(2s)$$
$$-\frac{1}{128}\cos(6r_1-6r_4)\cos(6r_2+2r_5)\cos(2r_3-2r_6)\sinh(z)\sinh(2s)$$
$$+\frac{27}{256}\cos(2r_1-2r_4)\cos(2r_2-2r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(2s)$$
$$-\frac{1}{256}\cos(2r_1-2r_4)\cos(2r_2+6r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(2s)$$
$$+\frac{3}{256}\cos(2r_1-2r_4)\cos(6r_2-6r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(2s)$$
$$-\frac{1}{256}\cos(2r_1-2r_4)\cos(6r_2+2r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(2s)$$
$$+\frac{1}{256}\cos(6r_1-6r_4)\cos(2r_2-2r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(2s)$$
$$+\frac{1}{256}\cos(6r_1-6r_4)\cos(2r_2+6r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(2s)$$
$$+\frac{1}{256}\cos(6r_1-6r_4)\cos(6r_2-6r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(2s)$$
$$+\frac{1}{256}\cos(6r_1-6r_4)\cos(6r_2+2r_5)\cos(2r_3-2r_6)\sinh(2z)\sinh(2s)$$
$$-\frac{1}{64}\cosh(2s)\cosh(2z)\cos(4r_1-4r_4)\cos(4r_3-4r_6)\sin(4r_5)\sin(4r_2$$

General pattern:

R symmetry = maximal compact subgroup $K(E_n) \subset E_n$

Bosons $\in$ single valued reps

Fermions $\in$ double valued (spinorial) reps

$D = 4$: $K(E_7) = SU(8) \subset E_{7(7)}$

Bosons : $1 \oplus 28 \oplus 70$ $\longrightarrow$ $SU(8)/\mathbb{Z}_2$

Fermions : $8 \oplus 56$

$D = 3$: $K(E_8) =$ "$SO(16)$" $\subset E_{8(8)}$

Bosons : 128_s

Fermions : $16_v \oplus 128_c$

NB : center of $Spin(16) = \mathbb{Z}_2^L \times \mathbb{Z}_2^R$ $\longrightarrow$ "$SO(16)$" $= Spin(16)/\mathbb{Z}_2^L$

(Keurentjes)

$\boxed{\text{Does this extend to } K(E_9) \text{ and } K(E_{10})\ ?}$

Fermions and maximal SUSY in $D=2$

$K(E_9) :=$ subgroup of $E_{9(9)}$ left invariant by generalized Chevalley involution

In terms of loop group realization:

$$\underset{\substack{\uparrow \\ \text{spectral} \\ \text{parameter}}}{\mathcal{U}(\gamma)} \in K(E_9) \Longleftrightarrow \underset{\substack{\uparrow \\ \in E_8(\mathbb{C}) \\ \text{for all } \gamma \in \mathbb{C}}}{\mathcal{U}(\gamma)} = \left[(\mathcal{U}^T)^{-1}\right]\left(\frac{1}{\gamma}\right)$$

$\Rightarrow$ at fixed points $\gamma = \frac{1}{\gamma} = \pm 1 \rightarrow SO(16)_\pm$

- $K(E_9)$ is <u>not</u> Kac-Moody (and $\neq \widehat{SO(16)}$)
- $K(E_9)$ is <u>not</u> simple (unlike $E_{9(9)}$): $\exists$ non-trivial ideals related to unfaithful spinorial representations

More specifically, $K(E_9)$ acts on chiral components of fermions via the "evaluation map" (Samtleben, N.)

$$\underset{\substack{\uparrow \\ 128_c}}{\chi'_\pm(x)} = \mathcal{U}(x,\gamma)\Big|_{\gamma=\pm 1} \chi_\pm(x)$$

looks like "generalized holonomy" group $SO(16)_+ \times SO(16)_-$ .....

... but action on full set of SUGRA fermions (and their derivatives) is more complicated, as it involves

$$\mathcal{U}^{(n)}(\gamma)\Big|_{\gamma=\pm 1} \quad \text{for } n \geq 1$$

Few results and many questions:

* $K(E_9)$ as a generalized holonomy

* Finite dimensional "generalized holonomy groups" from quotient algebras $K(E_9)/\mathcal{J}$ such that e.g. $\mathcal{J} = \mathcal{J}_{Dirac} \longleftrightarrow SO(16)_+ \times SO(16)_-$

* Faithful (hence, ∞-dimensional) spinorial representations of $K(E_9)$?

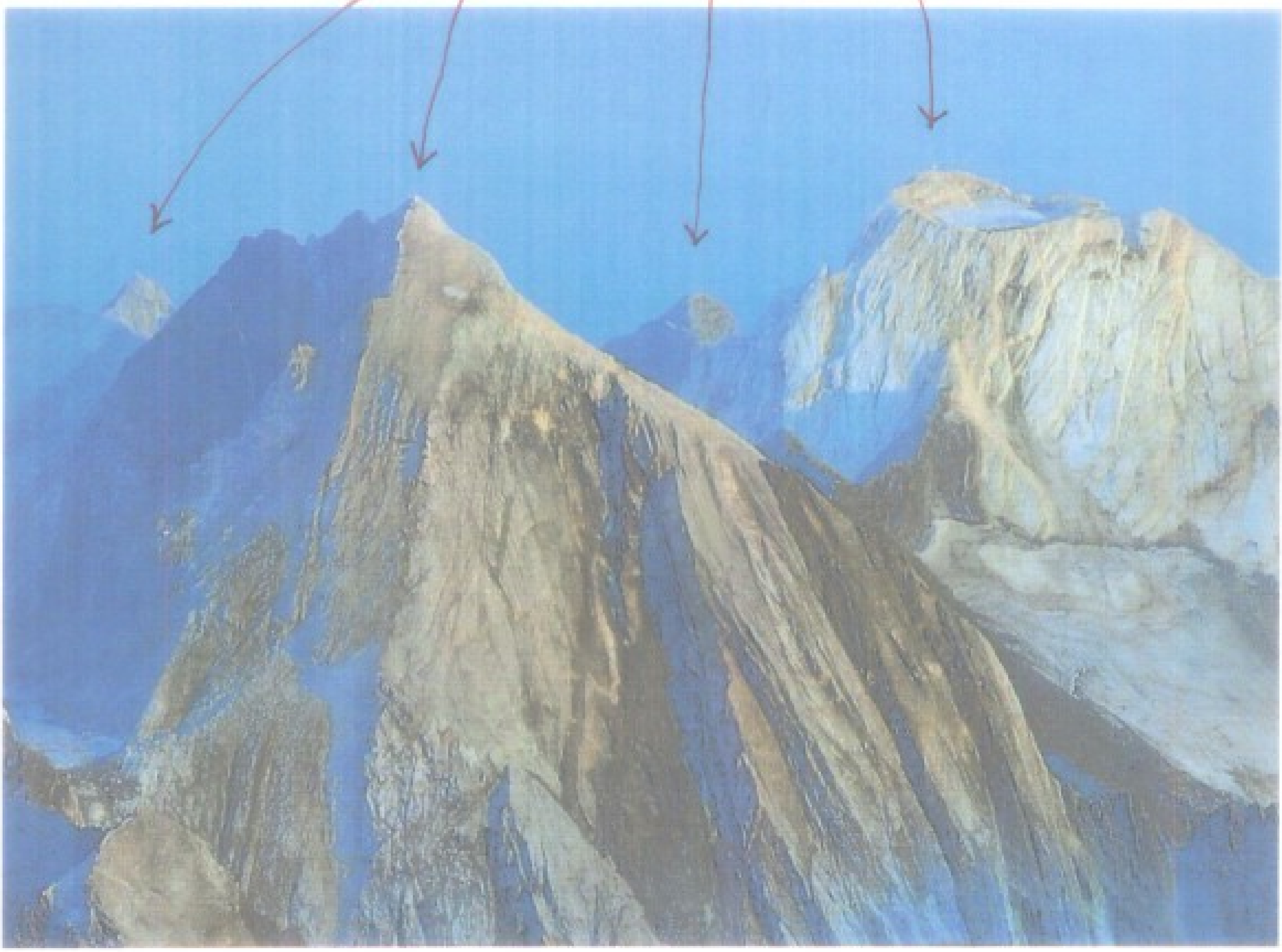
* $K(E_9)$ as a generalized R symmetry: does there exist an ∞-dimensional SUSY algebra with $K(E_9)$?

Idem for $K(E_{10}) \subset E_{10}$

... but much harder!

$\rightarrow$ DKN, dBHP

Maximal SUSY theories



the real (non-SUSY) world

Is there path leading from top to bottom, or vice versa?

Identify $SU(3) = [SU(3)_c \times SU(3)_f]_{diag}$

(M. Gell-Mann, '83

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix}_L \qquad 3_c \times \bar{3}_f \to 8 + 1 \qquad Q = \frac{2}{3} - q$$

$$\begin{pmatrix} \bar{u} \\ \bar{c} \\ \bar{t} \end{pmatrix}_L \qquad \bar{3}_c \times 3_f \to 8 + 1 \qquad Q = -\frac{2}{3} + q$$

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \qquad 3_c \times 3_f \to 6 + \bar{3} \qquad Q = -\frac{1}{3} + q$$

$$\begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix}_L \qquad \bar{3}_c \times \bar{3}_f \to \bar{6} + 3 \qquad Q = \frac{1}{3} - q$$

$$\begin{pmatrix} e^- \\ \mu^- \\ \tau^- \end{pmatrix}_L \qquad 1_c \times 3_f \to 3 \qquad Q = -1 + q$$

$$\begin{pmatrix} e^+ \\ \mu^+ \\ \tau^+ \end{pmatrix}_L \qquad 1_c \times \bar{3}_f \to \bar{3} \qquad Q = 1 - q$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L \qquad 1_c \times \bar{3}_f \to \bar{3} \qquad Q = -q$$

$$\begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \\ \bar{\nu}_\tau \end{pmatrix}_L \qquad 1_c \times 3_f \to 3 \qquad Q = +q$$

representations match with $N = 8$ for spurion charge $q = \frac{1}{6}$!

* Scheme is realized at $SU(3) \times U(1)$ stationary point of $N = 8$ SUGRA

(Warner, N., 1985)

... a **mirage?** ... but:

* Number of fermionic states in nature is $3 \times 16 = 48 = 56 - 8$ (so far...)

* Unbroken symmetry of low energy physics is $SU(3) \times U(1)$

* Analogous to Wilczek's scheme of color-flavor-locking

→ hep-ph/0003183

Remember Albert Einstein:

"The Lord is <u>subtle</u>, but not malicious"