Generalized N=2 Compactifications

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Introduction

String Theory: strings moving in 10d space-time background

$\Rightarrow$ contact with "our world": compactification

$\Rightarrow$ choose space-time background:

$$M_4 \times Y_6$$

M_4: 4 dim. space-time

Y_6: compact manifold $\Rightarrow$ determines amount of supersymmetry

$\Rightarrow$ fruitful interplay supersymmetry $\leftrightarrow$ geometry

recently: interplay supersymmetry $\leftrightarrow$ generalized geometry

purpose of this talk: review these developments for $N = 2$ theories

Type II string compactification:

Lorentz group of space-time background $M_{10} = M_4 \times Y_6$ decomposes

$$Spin(1,9) \to Spin(1,3) \times Spin(6)$$

spinors decompose accordingly: $\mathbf{16} \to (\mathbf{2}, \mathbf{4}) \oplus (\bar{\mathbf{2}}, \bar{\mathbf{4}})$

demand that two supercharges $Q^{1,2}$ exist $(N = 2)$

$\Rightarrow$ nowhere vanishing, globally defined spinor η needs to exist on Y_6

$\Rightarrow$ structure group of Y_6 has to be reduced

$$Spin(6) \to SU(3) \quad \text{s.t.} \quad \mathbf{4} \to \mathbf{3} + \mathbf{1}$$

$\Rightarrow$ Y_6 has $SU(3)$-structure [Gray, Hervella, Salamon, Chiossi, Hitchin, ...]

Supersymmetry of the background

gravitino transformation law

$$\delta\Psi = \nabla\eta + (\gamma \cdot F)\,\eta + \ldots\ , \qquad F = \text{background flux}$$

➪ supersymmetric background $\delta\Psi = 0$

- $F = 0$ and $\nabla\eta = 0$ $\Rightarrow$ Y_6 is Calabi-Yau manifold

  [Candelas,Horowitz,Strominger,Witten]

- $F \neq 0$ and $\nabla\eta \neq 0$ $\Rightarrow$ Y_6 has $SU(3)$ structure

  [Strominger, ...]

➪ non-supersymmetric backgrounds $\delta\Psi \neq 0$

$\Rightarrow$ $F \neq 0$ and/or $\nabla\eta \neq 0$ $\Rightarrow$ Y_6 has $SU(3)$ structure

$\Rightarrow$ analyze manifolds with flux and $SU(3)$ structure

Manifolds with $SU(3)$ structure:

characterized by two tensors J, Ω (follows from existence of η)

- $(1,1)$-form

$$J_{mn} = \eta^\dagger \gamma_{[m}\gamma_{n]}\eta \ , \qquad dJ \neq 0$$

$\Rightarrow$ almost complex structure

$$I_m{}^n = J_{mp} g^{pn} \ , \qquad I^2 = -1 \ , \qquad N(I) \neq 0$$

- $(3,0)$-form

$$\Omega_{mnp} = \eta^\dagger \gamma_{[m}\gamma_n\gamma_{p]}\eta \ , \qquad d\Omega \neq 0$$

- Remarks:
  - $dJ, d\Omega \ \sim$ (intrinsic) torsion of Y_6
  - manifolds are not complex, not Kähler, not Ricci-flat
  - manifolds are classified in terms of $SU(3)$ rep. of $dJ, d\Omega$
  - Calabi-Yau: $\nabla\eta = 0 \quad \Rightarrow \quad dJ = d\Omega = N(I) = 0$

$N = 2$ low energy effective action

$$\mathcal{S} = \int_{M_4} \tfrac{1}{2} R - g_{ab}(z)\, D_\mu z^a D^\mu z^b - V(z) + \dots$$

z^a: scalar fields

➪ g_{ab} metric on the scalar manifold $\mathcal{M} = \mathcal{M}^V_{\mathrm{SK}} \times \mathcal{M}^H_{\mathrm{QK}}$

$\mathcal{M}^V_{\mathrm{SK}}$: special Kähler manifold (vector multiplet sector)

$\mathcal{M}^H_{\mathrm{QK}}$: quaternionic Kähler manifold (hypermultiplet sector)

➪ V determined by Killing prepotential $\vec{P}$

next: compute $g_{ab}, \vec{P}$ for $SU(3)$ structure manifolds

compute g_{ab}

[Graña,Waldram,JL]

- decompose 10d fields under $SO(1,3) \times SU(3)$
- Impose "standard $N = 2$" (no massive gravitino multiplets)
  $\Rightarrow$ no $SU(3)$ triplets $\Rightarrow$ $d(J \wedge J) = 0$ and $d\Omega^{3,1} = 0$
- insert into $D = 10$ action:

$$\begin{aligned} S_{\mathsf{NS}} &= \int d^{10}x \sqrt{g}\, e^{-2\phi} \left[R + 4(\partial\phi)^2 - \tfrac{1}{12} H^2\right] \\ &= \int d^{10}x \sqrt{g^{(4)}} \left[R^{(4)} - 2(\partial\phi^{(4)})^2 - \tfrac{1}{12} e^{-4\phi^{(4)}} H_{(4)}^2 \right. \\ &\qquad \left. - \tfrac{1}{4} G^{mp} G^{nq} (\partial_\mu G_{mn} \partial^\mu G_{pq} + \partial_\mu B_{mn} \partial^\mu B_{pq}) + \ldots \right. \end{aligned}$$

where $\quad G_{\mu\nu}^{(4)} = e^{-2\phi_4} G_{\mu\nu}\,, \qquad \phi^{(4)} = \phi - \tfrac{1}{4} \ln\det G_{mn}$

last term: metric on the space of metric/B-field-deformations

compute g_{ab}

$SU(3)$ decomposition of metric

$$\delta G_{mn} = \left[\delta G_{mn}\right]_{\mathbf{8}} + \left[\delta G_{mn}\right]_{\mathbf{6}+\bar{\mathbf{6}}} = \delta J + \delta\Omega + \delta\bar{\Omega}$$

$\Rightarrow$ deformations form product of special geometries $\mathcal{M} = \mathcal{M}_J \times \mathcal{M}_\Omega$

with [Hitchin]

$$g_{ab} = \partial_a \partial_b (K_J + K_\Omega)\,, \qquad e^{-K_J} = \int_{Y_6} J \wedge J \wedge J\,, \qquad e^{-K_\Omega} = \int_{Y_6} \Omega \wedge \bar{\Omega}$$

Remarks:

- same as for Calabi-Yau manifolds [Strominger; Candelas, de la Ossa]
  (since $dJ, d\Omega$ do not appear)
- additional scalars/two-forms from RR-sector
  $\Rightarrow \mathcal{M} = \mathcal{M}^V_{SK} \times \mathcal{M}^H_{QK} \supset \mathcal{M}_J \times \mathcal{M}_\Omega$
- convenient formulation of $N = 2$: $\Rightarrow$ [de Wit, Samtleben, Trigiante]

compute $\vec{P}$

from supersymmetry transformation of gravitino

$$\delta\psi_{A\,\mu} = D_\mu \varepsilon_A + i\gamma_\mu S_{AB}\varepsilon^B + \ldots\ , \qquad S_{AB} = \tfrac{i}{2}\, e^{\frac{1}{2}K_V}\, \vec{\sigma}_{AB}\vec{P}\ , \qquad A = 1, 2$$

IIA:

$$P^1 + iP^2 \ = \ e^{\frac{1}{2}K_\Omega + \phi^{(4)}} \int_{Y_6} e^{-(B+iJ)} \wedge d\Omega\ , \qquad P^3 \ = \ e^{2\phi^{(4)}} \int_{Y_6} e^{-(B+iJ)} \wedge F_{\mathrm{A}}$$

IIB

$$F \equiv \textstyle\sum_{\text{RR-forms}} F^{\text{RR}}$$

$$P^1 + iP^2 = e^{\frac{1}{2}K_J + \phi^{(4)}} \int_{Y_6} \Omega \wedge d e^{-(B+iJ)}\ , \qquad P^3 = e^{2\phi^{(4)}} \int_{Y_6} \Omega \wedge F_{\mathrm{B}}$$

Remarks:

- potential: $V = V(\vec{P}, \partial\vec{P})$ (includes NS-flux H_3 & RR-fluxes F_A, F_B)
- torsion $d\Omega$, dJ appear in $\vec{P}$ but not in K

Manifolds with $SU(3) \times SU(3)$ structure:

In type II one can be more general: [Grana,Waldram,JL]

choose different spinors η^1, η^2 for the two gravitini $\Psi^{1,2}$

each spinor defines $SU(3)$ — together $SU(3) \times SU(3)$ structure

$SU(3) \times SU(3)$ structure: characterized by pair $J^{1,2}, \Omega^{1,2}$

more convenient formalism: define pure spinors Φ^+, Φ^- of $SO(6,6)$

$$\Phi^+ = e^B \eta^1_+ \otimes \bar{\eta}^2_+ = \sum \Phi^+_{\text{even}} \ , \qquad \Phi^- = e^B \eta^1_+ \otimes \bar{\eta}^2_- = \sum \Phi^+_{\text{odd}} \ ,$$

$SU(3)$ structure ($\eta^1 = \eta^2$):

$$\Phi^+ = e^{B+iJ} \ , \qquad \Phi^- = \Omega \ ,$$

Couplings for $SU(3) \times SU(3)$ compactifications

- decompose 10d fields under $SO(1,3) \times SU(3) \times SU(3)$
- impose "standard $N = 2$" (no massive gravitino multiplets)
  $\Rightarrow$ no $(\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3})$
- insert into $D = 10$ action: $\mathcal{M} = \mathcal{M}_{\Phi^+} \times \mathcal{M}_{\Phi^-}$

with $e^{-K_{\Phi^+}} = \int_{Y_6} \langle \Phi^+, \overline{\Phi}^+ \rangle \,, \qquad e^{-K_{\Phi^-}} = \int_{Y_6} \langle \Phi^-, \overline{\Phi}^- \rangle$

where $\langle \Phi^+, \overline{\Phi}^+ \rangle = \Phi_0^+ \wedge \overline{\Phi}_6^+ - \Phi_2^+ \wedge \overline{\Phi}_4^+ + \Phi_4^+ \wedge \overline{\Phi}_2^+ - \Phi_6^+ \wedge \overline{\Phi}_0^+$, etc.

$$\text{IIA:} \quad P^1 + iP^2 = e^{\frac{1}{2}K_{\Phi^-} + \phi^{(4)}} \int_{Y_6} \langle \Phi^+, d\Phi^- \rangle \,, \qquad P^3 = e^{2\phi^{(4)}} \int_{Y_6} \langle \Phi^+, F_{\mathsf{A}} \rangle$$

$$\text{IIB:} \quad P^1 + iP^2 = e^{\frac{1}{2}K_{\Phi^+} + \phi^{(4)}} \int_{Y_6} \langle \Phi^-, d\Phi^+ \rangle \,, \qquad P^3 = e^{2\phi^{(4)}} \int_{Y_6} \langle \Phi^-, F_{\mathsf{B}} \rangle$$

Mirror symmetry

➪ for Calabi-Yau manifolds:

'every' Y has a mirror manifold $\tilde{Y}$ with

$$h^{1,1}(Y) = h^{1,2}(\tilde{Y}) , \qquad h^{1,2}(Y) = h^{1,1}(\tilde{Y}) ,$$

$$\mathcal{M}_J(Y) \equiv \mathcal{M}_\Omega(\tilde{Y}) , \qquad \mathcal{M}_\Omega(Y) \equiv \mathcal{M}_J(\tilde{Y}) .$$

manifestation in string theory:

IIA in background $R_{1,3} \times Y$ $\equiv$ IIB in background $R_{1,3} \times \tilde{Y}$

useful for computing instanton correction to the large volume limit

Mirror symmetry with fluxes and generalized geometries

[Grana,Minasian,Petrini,Tomasiello; Gurrieri,Micu,Waldram,JL; Grana,Waldram,JL]

so far only in the large volume (supergravity) limit

$$\mathcal{M}_{\Phi^+}(Y) \equiv \mathcal{M}_{\Phi^-}(\tilde{Y}) , \qquad \mathcal{M}_{\Phi^-}(Y) \equiv \mathcal{M}_{\Phi^+}(\tilde{Y}) , \qquad \vec{P}(Y) \equiv \vec{P}(\tilde{Y}) ,$$

for

$$\Phi^+(Y) = \Phi^-(\tilde{Y}) , \qquad \Phi^-(Y) = \Phi^+(\tilde{Y}) , \qquad F_A(Y) = F_B(\tilde{Y})$$

with

$$\mathrm{d}\,\mathrm{Im}\Phi^- = 0$$

$\Rightarrow$ (generalized) half-flat geometry

$\Rightarrow$ type IIA and type IIB compactification are equivalent !

Non-geometric backgrounds

[Dabholkar,Hull; Mathai,Rosenberg; Shelton,Taylor,Wecht; Grange,Schäfer-Nameki]

idea:

Y_6 can be a T-fold (structure group includes T-duality)

- is well defined in string theory
- arises as mirror dual of certain class of fluxes (magnetic fluxes)
- can be discussed in terms of $SU(3) \times SU(3)$ formalism developed

[Grana,Waldram,JL]

conjecture:

Non-geometric backgrounds can also be classified in terms of $SU(3) \times SU(3)$ structure

$\Rightarrow$ G-structures can be extended to non-geometrical backgrounds

Non-perturbative dualities with flux

[Curio,Klemm,Körs,Lüst; Micu,JL]

Non-perturbative duality:

Heterotic on $K3 \times T^2$ $\leftrightarrow$ Type IIA on Calabi-Yau threefold

➪ Heterotic: gauge-field flux on $K3$:

$$\int_{\gamma^i} F^A = \theta^{Ai} \,, \qquad i = 1, \ldots, 22$$

internal b becomes charged and potential is induced

$$Db^i = db^i - \theta^i_A A^A \,, \qquad V_{\text{het}} \neq 0$$

➪ proposed dual: type IIA on specific $SU(3)$-structure manifold

checks: Killing vectors, consistency of V with gauged supergravity

problem: duality in hypermultiplet sector!

Rewrite ten-dimensional supergravity in $N=2$-form [de Wit,Nicolai]

- decompose 10d fields under $SO(1,3)\times SU(3)\times SU(3)$
- impose "standard $N=2$" (no massive gravitino multiplets)
  $\Rightarrow$ no $(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{3})$
  $\Rightarrow$ ten-dimensional fields fall into $N=2$ multiplets of $SO(1,3)$
- insert into ten-dimensional action but do not integrate over Y_6
- define space of metric deformations as before
- resulting geometries are again special Kähler geometries with

$$e^{-K_J}=J\wedge J\wedge J\ ,\qquad e^{-K_\Omega}=\Omega\wedge\bar{\Omega}$$

$$\mathrm{IIA:}\quad P^1+iP^2 = e^{\frac{1}{2}K_\Omega+\phi^{(4)}}e^{-(B+iJ)}\wedge d\Omega\ ,\qquad P^3 = e^{2\phi^{(4)}}e^{-(B+iJ)}\wedge F_{\mathsf{A}}$$

$$\mathrm{IIB:}\quad P^1+iP^2=e^{\frac{1}{2}K_J+\phi^{(4)}}\Omega\wedge de^{-(B+iJ)}\ ,\qquad P^3=-e^{2\phi^{(4)}}\Omega\wedge F_{\mathsf{B}}$$

Summary

- compactifications on manifolds with $SU(3) \times SU(3)$ structure
  - $\Rightarrow$ product of special geometries
  - $\Rightarrow$ $K = K_{\Phi^+} + K_{\Phi^-}$ is independent on torsion
- potential depends on torsion and background fluxes
- mirror symmetry and (some) non-perturbative dualities hold
  (in the supergravity limit)
- non-geometric backgrounds can be included
  in the $SU(3) \times SU(3)$ formalism
- landscape seems even richer than previously thought