BPS multiplets, supersymmetry breaking and non-perturbative effects

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I have been one of the users of supergravity in my scientific life, but I have never inhaled.

I have never met J. Scherk, but I have worked on two (of the many) aspects of his mark in theoretical physics:

♠ Anti-gravity, aka BPS states.

♠ Geometrical Supersymmetry breaking (as originally conceived by J. Scherk and J. Schwarz).
The Plan

- Ouverture
- The supersymmetry algebra and BPS states
- BPS saturated couplings and helicity supertraces
- BPS mass formulae: the $\mathcal{N} = 4$ paradigm.
- Scherk-Schwarz supersymmetry breaking in perturbation theory
- Duality and Scherk-Schwarz supersymmetry breaking beyond perturbation theory
- Epilogue

BPS multiplets ...
Supersymmetry has spread its magic over the last 40 decades.

It quickly met, head on, gravity to form supergravity and raise the stakes.

Supergravity quickly teamed up with string theory in an unparalleled match to dominate theoretical attempts at unification in the last two decades.

The Supergravity/String theory match was turbulent at first. String theorists looked with suspicion or disdain at supergravity. Supergravity theorists felt for a while out of place and time.

It is supersymmetry and the need to understand strong coupling limits that provided a meeting point, and since the mid-nineties the two directions go hand-in-hand.

It is the meeting point, via BPS multiplets, supersymmetry and its breaking, and non-perturbative effects, that I will review here. The review will be short, and dominated by the things I understand best.

BPS multiplets ...
The supersymmetry algebra and BPS states

• The key to BPS multiplets and their magic lies in the special properties of the supersymmetry algebra.

\[
\{Q^I_{\alpha}, Q^J_{\beta}\} = \epsilon_{\alpha\beta} Z^{IJ}, \quad \{\bar{Q}^I_{\dot{\alpha}}, Q^J_{\dot{\beta}}\} = \epsilon_{\dot{\alpha}\dot{\beta}} \bar{Z}^{IJ}, \quad \{Q^I_{\alpha}, \bar{Q}^J_{\dot{\alpha}}\} = \delta^{IJ} 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu
\]

\(I = 1, 2, \ldots, N\).

• Invariance under the \(U(N)\) \(R\)-symmetry that rotates \(Q, \bar{Q}\).

• **Massive representations.** Go to the rest frame \(P \sim (M, \vec{0})\).

\[
\{Q^I_{\alpha}, \bar{Q}^J_{\dot{\alpha}}\} = 2M \delta^{I,J} \quad \{Q^I_{\alpha}, Q^J_{\beta}\} = \{\bar{Q}^I_{\dot{\alpha}}, \bar{Q}^J_{\dot{\beta}}\} = 0
\]

• Define the \(2N\) fermionic harmonic creation and annihilation operators

\[
A^I_{\alpha} = \frac{1}{\sqrt{2M}} Q^I_{\alpha}, \quad A^\dagger_I_{\dot{\alpha}} = \frac{1}{\sqrt{2M}} \bar{Q}^I_{\dot{\alpha}}
\]

• Start with the Clifford vacuum \(|\Omega\rangle\), (annihilated by the \(A^I_{\alpha}\)) and act with \(A^\dagger_I_{\dot{\alpha}}\).

\[
\text{total number of states} = \sum_{n=0}^{2N} \binom{2N}{n} = 2^{2N}
\]

• **Massless representations.** Go to the frame \(P \sim (-E, 0, 0, E)\)

\[
\{Q^I_{\alpha}, \bar{Q}^J_{\dot{\alpha}}\} = 2 \begin{pmatrix} 2E & 0 \\ 0 & 0 \end{pmatrix} \delta^{IJ}
\]
\( Q_2^I, \bar{Q}_2^I = 0 \) for such reps. The representation is \( 2^N \)-dimensional. This is a trivial example of a BPS rep.

- **Non-zero central charges.** Skew-diagonalize \( Z^{IJ} \) with real positive skew-eigenvalues \( Z_m \).

\[
\{ Q^{am}_\alpha, \overline{Q}^{bn}_{\dot{\alpha}} \} = 2M \delta^{\alpha_{\dot{\alpha}}} \delta^{ab} \delta^{mn}, \quad \{ Q^{am}_\alpha, Q^{bn}_{\beta} \} = Z_n \epsilon^{\alpha\beta} \epsilon^{ab} \delta^{mn}
\]

- Define

\[
A^m_\alpha = \frac{1}{\sqrt{2}}[Q^1_\alpha + \epsilon_{\alpha\beta}Q^2_{\beta}], \quad B^m_\alpha = \frac{1}{\sqrt{2}}[Q^1_\alpha - \epsilon_{\alpha\beta}Q^2_{\beta}]
\]

\[
\{ A^m_\alpha, A^n_{\dot{\beta}} \} = \delta_{\alpha\dot{\beta}} \delta^{mn}(2M + Z_n), \quad \{ B^m_\alpha, B^n_{\dot{\beta}} \} = \delta_{\alpha\dot{\beta}} \delta^{mn}(2M - Z_n)
\]

- From Unitarity → Bogomolnyi bound

\[
M \geq \max \left[ \frac{Z_m}{2} \right]
\]

- Assume \( 0 \leq r \leq N/2 \) of the \( Z_m = 2M \). Then \( 2r \) of the \( B \)-oscillators vanish identically. We are left with \( 2N - 2r \) creation and annihilation operators. The representation has \( 2^{2N-2r} \) states. The maximal case, has as many states as the massless multiplet.
Central charges depend on couplings and vevs.

Massive BPS states can become massless without interference from other multiplets. The inverse is also true.

Massive BPS states are absolutely stable in large regions of the moduli space. They can be reliably extrapolated at strong coupling.

There are special effective field theory couplings that obtain quantum corrections only from BPS states. They are termed “BPS-saturated couplings”. They have special properties, but they typically include the lowest energy relevant couplings.

Helicity supertrace formulae are at the heart of the connection between BPS states and quantum corrections to effective couplings.

All successes of duality conjectures and non-perturbative determinations of EFTs à la Seiberg-Witten rely on BPS-saturated couplings.
Helicity supertraces

The role of helicity supertraces in $\beta$-function calculations and in issues of supersymmetry breaking is known since 1981 \cite{Curtright-81, Ferrara+Savoy+Girardello-81}

It was realized that they are central in quantitative tests of duality conjectures \cite{Bachas+Kiritsis-96, Gregori+Kiritsis+Kounnas+Obers+Petropoulos+Pioline-97, Bachas+Fabre+Kiritsis+Obers+Vanhove-97, Kiritsis+Obers-97, Gregori+Kounnas+Petropoulos-98}

In 4d they are defined as

$$B_{2n}(R) = \text{Tr}_R[(-1)^{2\lambda} \lambda^{2n}]$$

The “helicity-generating function” of a given supermultiplet $R$

$$Z_R(y) = \text{Str} \ y^{2\lambda}.$$  

For a particle of spin $j$ we have

$$Z[j] = (-)^{2j}\left(\frac{y^{2j+1} - y^{-2j-1}}{y - y^{-1}}\right) \quad \text{massive}, \quad Z[j] = (-)^{2j}(y^{2j} + y^{-2j}) \quad \text{massless}$$

The supertrace of the $n$-th power of helicity can be extracted from the generating functional through

$$B_n(R) = (y^2 \frac{d}{dy^2})^n Z_R(y)|_{y=1}$$

An $\mathcal{N} = 2$ example: This is relevant for the two derivative N=2 effective action

$$B_0(\text{any}) = 0, \quad B_2(M_\lambda) = (-1)^{2\lambda+1}, \quad B_0(S_j) = (-1)^{2j+1}j(j+1), \quad B_2(M_j) = 0$$

For $\mathcal{N} = 4$ $B_0 = B_2 = 0, B_4(L) = B_6(L) = 0, B_4(I) = 0$

BPS multiplets ...,
The BPS mass formula: the $\mathcal{N}=4$ paradigm

- We consider $\mathcal{N}=4$ supergravity coupled to $n$ vector multiplets. We take $n=22$, relevant to the heterotic/$T^6$ compactifications or type II/$K3 \times T^2$

- The scalar space is $SU(1,1)/U(1) \times O(6,22)/(O(6) \times O(22))$. The first part is parameterized by the complex $S$ field while the second by an $O(6,22)$ symmetric matrix $M$,

$$M^T = M , \quad M^TLM = L , \quad L = \begin{pmatrix} 0 & 1_6 & 0 \\ 1_6 & 0 & 0 \\ 0 & 0 & -1_{16} \end{pmatrix}$$

In the heterotic theory it is a function of the internal components of $G, B, A^I$

$$S = S_1 + i S_2 = a + i e^{-2\phi}$$

- The gauge and scalar action is

$$\mathcal{L} \sim \left[ -4 \frac{\partial^\mu S \partial^\nu \bar{S}}{\text{Im} S^2} - 2\text{Im} S (M^{-1})_{ij} F_{\mu \nu}^i F_{j,\mu \nu} + 2\text{Re} S L_{ij} F^i \wedge F^j + \text{Tr}(\partial^\mu M \partial^\mu M^{-1}) \right]$$
There is a perturbative invariance under $O(6,22,Z)$ transformations

$$M \to \Omega \, M \Omega^T \ , \quad F_{\mu\nu} \to \Omega \, F_{\mu\nu} \ , \quad \Omega \in O(6,22,Z)$$

with electric charges transforming as

$$e_i \to \Omega_{ij} e_j$$

- Finally there is electric magnetic duality

$$S \to \frac{aS + b}{cS + d} \ , \quad M \to M \ , \quad F^i_{\mu\nu} \to (c \, \text{Re}S + d) F^i_{\mu\nu} + c \, \text{Im}S \, (ML)_{ij}^* F^j_{\mu\nu}$$

- We parametrize the electric and magnetic charges of generic dyons

$$\tilde{Q}_e = \frac{1}{\sqrt{2} \, \text{Im}S} M (\tilde{\alpha} + \text{Re}S \, \tilde{\beta}) \ , \quad \tilde{Q}_m = \frac{1}{\sqrt{2} \, \text{Im} \beta} \ , \quad \tilde{\alpha}, \tilde{\beta} \in \mathbb{Z}^{28}$$

so that they satisfy the Dirac-Schwinger-Zwanziger-Witten quantization condition.
• The BPS mass formula can be expressed in two equivalent ways

\[ M_{BPS}^2 = \frac{\text{Im} S}{4} \left[ Q_e^t \tilde{M}_+ Q_e + Q_m^t \tilde{M}_+ Q_m + 2\sqrt{(Q_e^t \tilde{M}_+ Q_e)(Q_m^t \tilde{M}_+ Q_m) - (Q_e^t \tilde{M}_+ Q_m)^2} \right] \]

\[ = \frac{1}{4 \text{Im} S}(\alpha^t + S\beta^t)M_+(\alpha + \bar{S}\beta) + \frac{1}{2} \sqrt{(\alpha^t M_+ \alpha)(\beta^t M_+ \beta) - (\alpha^t M_+ \beta)^2} \]

with \( M_+ = M + L \) and \( \tilde{M}_+ = LM_+L \).

• It is \( O(6,22,\mathbb{Z}) \) invariant

\[ \alpha \rightarrow \Omega \alpha , \quad \beta \rightarrow \Omega \beta , \]

and \( SL(2,\mathbb{Z})_S \) invariant

\[ \begin{pmatrix} \bar{\alpha} \\ \bar{\beta} \end{pmatrix} \rightarrow \begin{pmatrix} a & -b \\ -c & d \end{pmatrix} \begin{pmatrix} \bar{\alpha} \\ \bar{\beta} \end{pmatrix} \]

• The square-root factor is proportional to the difference of the two central charges squared.
• It is zero when $\vec{\beta} \sim \alpha$ and we have 1/2 BPS multiplets. Otherwise we have 1/4 BPS multiplets.

• For perturbative BPS states of the heterotic string, $\vec{\beta} = 0$.

$$M_{\text{BPS,pert}}^2 = \frac{1}{4 \ ImS} \alpha^t \cdot M_+ \cdot \alpha = \frac{1}{4 \ ImS} p_L^2$$

• The multiplicity of 1/2 perturbative heterotic BPS states is given by the $B_4$ helicity formula:

$$d_N \text{ states with } N = \frac{1}{2} \alpha^t \cdot L \cdot \alpha \quad , \quad \frac{1}{\eta^{24}} = \sum_{N=-1}^{\infty} d_N \, q^N$$

• The non-perturbative multiplicity is conjectured to be

$$d_{N_e,N_m,N_s} \text{ states with } N_e = \frac{1}{2} \alpha^t \cdot L \cdot \alpha \quad , \quad N_m = \frac{1}{2} \beta^t \cdot L \cdot \beta \quad , \quad N_s = \alpha^t \cdot L \cdot \beta$$

$Dijkgraaf+Verlinde+Verlinde-96$
\[
\frac{1}{\Phi(\Omega)} = \sum_{N_e, N_m, N_s} d_{N_e, N_m, N_s} e^{-2\pi i (N_e \rho + N_m \tau + N_s v)} , \quad \Omega = \begin{pmatrix} \rho & v \\ v & \tau \end{pmatrix}
\]

- It correctly accounts for the associated black-hole entropy of dyonic branes (1/4 BPS).

\[
S = \pi \sqrt{(\alpha^t \cdot L \cdot \alpha)(\beta^t \cdot L \cdot \beta) - (\alpha^t \cdot L \cdot \beta)^2}
\]

- Recently, even the 1/2 BPS black-holes have been accounted by the perturbative multiplicity formula.

\[Dabholkar-04, Sen-05, Dabholkar+Denef+Moore+Pioline-05\]
An $N=2$ truncation

- We may freeze the $(4,22)$ moduli, and focus on the remaining $(2,2+16)$ associated with a single $T^2$.

- This is also an $\mathcal{N}=2$ truncation, appropriate for $K3 \times T^2$ compactifications.

- The $T, U$ and $W^i$ moduli are defined as

$$G = \frac{T_2 - \frac{1}{2U_2} \sum_i (\text{Im}W_i)^2}{U_2} \begin{pmatrix} 1 & U_1 \\ U_1 & |U|^2 \end{pmatrix}, \quad B = \left( T_1 - \frac{\sum_i \text{Re}W_i \text{Im}W_i}{2U_2} \right) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$W_i = -Y_2^i + UY_1^i, \quad K = -\log[T_2U_2 - \frac{1}{2} \sum_i \text{Im}W_i^2]$$
The BPS mass formula now is

\[ M_{BPS}^2 = \frac{-m_1 U + m_2 + T n_1 + (T U - \frac{1}{2} \sum_i W_i^2) n_2 + W_i q^i + \sum_i W_i^2 (T U - \frac{1}{2} \sum_i \text{Im} W_i^2)}{4 S_2 \left( T_2 U_2 - \frac{1}{2} \sum_i \text{Im} W_i^2 \right)} \]

\[ S \left[ -\tilde{m}_1 U + \tilde{m}_2 + T \tilde{n}_1 + \tilde{n}_2 (T U - \frac{1}{2} \sum_i W_i^2) + \tilde{q}^i W_i \right] \]

\[ + \frac{2}{4 S_2 \left( T_2 U_2 - \frac{1}{2} \sum_i \text{Im} W_i^2 \right)} \]

- Invariant under \( SL(2, \mathbb{Z}) \times O(2, 18, \mathbb{Z}) \)
- \( O(2, 18, \mathbb{Z}) \) acts in the standard fashion. \( SL(2, \mathbb{Z}) \) acts by interchanging electric and magnetic charges.

\[ S \rightarrow -\frac{1}{S} \quad , \quad \begin{pmatrix} m_i \\ n_i \\ q_i \\ \tilde{m}_i \\ \tilde{n}_i \\ \tilde{q}_i \end{pmatrix} \rightarrow \begin{pmatrix} \tilde{m}_i \\ \tilde{n}_i \\ \tilde{q}_i \\ -m_i \\ -n_i \\ -q_i \end{pmatrix} \quad , \quad S \rightarrow S + 1 \quad , \quad \begin{pmatrix} m_i \\ n_i \\ q_i \\ \tilde{m}_i \\ \tilde{n}_i \\ \tilde{q}_i \end{pmatrix} \rightarrow \begin{pmatrix} m_i - \tilde{m}_i \\ n_i - \tilde{n}_i \\ q_i - \tilde{q}_i \\ \tilde{m}_i \\ \tilde{n}_i \\ \tilde{q}_i \end{pmatrix} \]

BPS multiplets ...
Heterotic/type-II duality and branes

- heterotic$/T^6$ is non-perturbatively equivalent to type II$/K3 \times T^2$.

The simple tests:

- The non-perturbative states (monopoles) on heterotic side are fundamental particles on type II and vice versa. The heterotic NS5-brane on $T^4$ is dual to the perturbative type IIA string and vice versa. \textit{Hull+Townsend-94, Witten-95, M. Duff-95}

- The charged states $q^i \neq 0$ are RR states in the type II side. The non-abelian completion comes from the type-II D-branes. \textit{Hull+Townsend-94, Witten-95}

- BPS multiplicities agree. \textit{Bershadsky+Sadov+Vafa-96}

- The $F^4$ thresholds on the two side quantitatively agree (One-loop $\leftrightarrow$ tree level). \textit{Kiritsis+Obers+Pioline-00}

- Indirect tests: upon reduction of susy: heterotic on $K3 \times T^2 \sim$ type-II on CY (elliptic K3 fibration). \textit{Kachru+Vafa-95, Ferrara+Harvey+Strominger+Vafa-95}

BPS multiplets ..., \textit{E. Kiritsis}
Matching the type-II BPS spectrum

The key: emergence of massless vectors

- $C_3$ and $C_1$ on K3, give rise to (4,20) vectors, matching the Hodge diamond of K3.

- The charged states are: D4 branes wrapped on K3, D0 branes and D2 branes wrapped on the (3,19) cycles dual to the two-forms. This generates the $\Gamma_{4,20}$ lattice where the integers are wrapping and multiplicity numbers for the D-branes.

Upon descent to 4 dimensions we have $\Gamma_{2,2} \times \Gamma_{4,20} \to \Gamma_{6,22}$ There for (2,2) from the vectors are NS-NS while the rest (4,20) are RR.
• On the heterotic side, the magnetic states arise from the NS5-brane. Wrapped around $T^6$ it gives rise to a magnetic copy of the electric spectrum. The associated masses scale according to the BPS formula as

$$|S|^2/S_2 \sim \frac{1}{g_s^2}$$

• On the type-II side, the missing states are also generated by the NS5 branes wrapped around $K3 \times T^2$.

• Heterotic/type II duality involves

$$S \leftrightarrow T$$

and a dualization of the NS two-form that translates into:

$$\begin{pmatrix} m_1 \\ m_2 \\ n_1 \\ n_2 \\ q_i \end{pmatrix} \rightarrow \begin{pmatrix} m_1 \\ m_2 \\ \tilde{m}_2 \\ -\tilde{m}_1 \\ q_i \end{pmatrix}, \quad \begin{pmatrix} \tilde{m}_1 \\ \tilde{m}_2 \\ \tilde{n}_1 \\ \tilde{n}_2 \\ \tilde{q}_i \end{pmatrix} \rightarrow \begin{pmatrix} -n_2 \\ n_1 \\ \tilde{n}_1 \\ \tilde{n}_2 \\ \tilde{q}_i \end{pmatrix}$$

• These transformations are valid, even if we include the (4,4) part of the lattice.
Spontaneous supersymmetry breaking (à la Scherk-Schwarz)

- The Scherk-Schwarz Susy breaking has been successfully implemented in string theory:
  
  * Rohm-84, Kounnas+Porrati-88, Ferarra+Kounnas+Porrati+Zwirner-89, Kounnas+Rostand-90

- It has been shown to be generated by appropriate freely-acting orbifolds. 
  
  * Kiritsis+Kounnas-97

- We will study here first the $\mathcal{N}=4 \rightarrow \mathcal{N}=2$ partial breaking in the toroidal heterotic string. We will then use BPS data and duality to construct non-perturbative models of susy breaking.
  
  * Kiritsis+Kounnas+Petropoulos+Rizos-96-98

- To start, we choose moduli to factorize the lattice

  \[ \Gamma_{6,22} \rightarrow \Gamma_{2,18} \times \Gamma_{4,4} \]
• We consider an orbifold that acts as a $\mathbb{Z}_N$ rotation on $\Gamma_{4,4}$ and by a $\mathbb{Z}_N$ translation $\epsilon/N$, with $\epsilon \equiv (\epsilon_L, \epsilon_R, \zeta)$ on $\Gamma_{2,18}$. The rotation breaks half of the supersymmetry.

• Modular invariance implies

$$\frac{\epsilon^2}{2} \equiv \epsilon_L \cdot \epsilon_R - \frac{1}{2} \zeta \cdot \zeta \equiv 1 \text{ mod } N^2$$

• T-duality acts both on moduli $(T, U, W_i)$ and the shift vector $\epsilon$. $\epsilon^2$ is T-duality invariant.

• The mass of the two gravitini:

$$M_{\text{gravitino}}^2 = \left| -m_1 U + m_2 + T n_1 + (T U - \frac{1}{2} \sum_i W_i^2) n_2 + W_i q^i \right|^2$$

$$\frac{4 S_2 (T_2 U_2 - \frac{1}{2} \sum_i \text{Im} W_i^2)}{4 S_2 (T_2 U_2 - \frac{1}{2} \sum_i \text{Im} W_i^2)}$$

$$m \cdot \epsilon_R + n \cdot \epsilon_L - q \cdot \zeta \equiv \pm 1 \text{ mod } N$$

• Many solutions, but the two lightest ones are the massive gravitini. The rest are KK descendants. This depend on the region of moduli space.

• The massive gravitons are BPS states of the unbroken $\mathcal{N}=2$ supersymmetry.

• The full BPS spectrum (half-BPS vector multiplets and hypermultiplets can be easily computed via the $B_2$ helicity supertrace.)
It is given by the O(2,18) BPS mass formula. In the “untwisted” sector

\[ M_{\text{BPS}}^2 = \frac{|-m_1 U + m_2 + T n_1 + (T U - \frac{1}{2} \sum_i W_i^2) n_2 + W_i q^i|^2}{4 S_2 (T U_2 - \frac{1}{2} \sum_i \text{Im} W_i^2)} \]

They are hypermultiplets when

\[ m \cdot \epsilon_R + n \cdot \epsilon_L - q \cdot \zeta = \text{even} \]

and vector-like multiplets when

\[ m \cdot \epsilon_R + n \cdot \epsilon_L - q \cdot \zeta = \text{odd} \]

In the twisted sector

\[ (m, n, q) \rightarrow \left( m + \frac{\epsilon_L}{2}, n + \frac{\epsilon_R}{2}, q + \frac{\zeta}{2} \right) \]
We would like to compare spontaneous SUSY breaking à la SS and "hard" susy breaking.

Consider a $Z_2$ orbifold action that breaks susy. All massless gravitini are removed from the spectrum. The spectrum up to $M_s$ is non-supersymmetric.

Compare with a similar orbifold where the $Z_2$ action is accompanied by a $Z_2$ shift on a circle of radius $R$. Here the gravitini have masses $\sim 1/R$.

When $R \to \infty$ supersymmetry is restored. $R > 1$ is the softly broken region.

When $R \to 0$, the massive gravitini decouple and the vacuum is identical to the hard susy-breaking one (provided susy is not completely broken).

BPS multiplets ...
By appropriate non-free action we may break susy in various other ways:

♠ \( \mathcal{N} = 4 \rightarrow \mathcal{N} = 1 \)

♠ \( \mathcal{N} = 4 \rightarrow \mathcal{N} = 0 \)

By accompanying these actions with lattice shifts we obtain vacua with various type of partial supersymmetry breaking.

Simple illustrative Examples:

• \( \mathcal{N} = 4 \rightarrow \mathcal{N} = 0 \): Heterotic string in finite temperature: \((-1)^F\) plus \(\mathbb{Z}_2\) translation along Euclidean time.

  Rohm-84, Attick+Witten-88
  Kounnas+Ronstant-90
  Antoniadis+Derendiger+Kounnas-98

• \( \mathcal{N} = 4 \rightarrow \mathcal{N} = 1 \): \(\mathbb{Z}_2 \times \mathbb{Z}_2\) action on \(T^2 \times T^2 \times T^2\) with a translation in the transverse torus.

  Kiritsis+Kounnas-97, Gregori+Kounnas+Rizos-99

BPS multiplets ...
(Internal) Fluxes and supersymmetry breaking
(the prehistory)

• It was realized early on this supersymmetry breaking could be interpreted as due to special discrete internal fluxes

  Kounnas-87, Kiritsis+Kounnas-97

• The (discrete) perturbations have the form

\[ \Delta S \sim \int d^2z \ F_{ij}^a \ [\psi^i \psi^j - X^{(i} \partial X^{j)}] \ J^a \]

They are constrained (quantized) by requiring preservation of world-sheet susy

• The appropriate discrete operator

\[ U = e^{\oint dz \ F_{ij}^a \ [\psi^i \psi^j - X^{(i} \partial X^{j)}]} \]

can be constructed by fermionization of \( X^I \).

• They generate non-vanishing auxiliary fields in 4d.
For example the $\mathcal{N} = 4 \rightarrow \mathcal{N} = 2$ breaking is due to a self-dual “flux”, $F_{34} = -F_{55}$.

- Their type-II duals correspond to special points of flux compactifications with both RR and NS fluxes turned on.  

  Kiritsis+Kounnas-97

Since then, several advances:

- Superpotentials have been advanced to deal with fluxes  

  Taylor+Vafa-99, Gukov+Witten+Vafa-01

- The effects of non-trivial fluxes in the type II context was resurrected with different motivation (warping) several years later  

  Giddings+Kachru+Polchinski-01

- Supersymmetry breaking was rediscussed in this framework, together with the important issue of moduli stabilisation.  

  Kachru+Schultz+Trivedi-02  
  Kachru+Kallosh+Linde+Trivedi-03

BPS multiplets ...,
Duality and non-perturbative supersymmetry breaking

- We consider for simplicity a heterotic $Z_2$ freely-acting orbifold with a shift vector $\epsilon$ acting on $\Gamma_{2,18}$.

- According to the “adiabatic argument” a similar action on the type II side will give a dual ground-state.

  Vafa+Witten-95

  The $Z_2$ rotation acts as $(-1)^F_L$ accompanied by an involution $e$ that changes the sign of 16 of the self-dual two-forms on K3.

- The $\Gamma_{2,18}$ (electric) shift vector $\epsilon = (\epsilon_L, \epsilon_R, \zeta)$ gives $Z_2$ phases to various states:

- The (0,16) shift can be associated with (discrete) RR fluxes threading the appropriate cycles of K3.
• To see further the correspondence, I recall the heterotic/type-II map on $T^2$ charges:

$$ T \leftrightarrow S \ , \ \begin{pmatrix} m_1 \\ m_2 \\ n_1 \\ n_2 \\ q_i \end{pmatrix} \rightarrow \begin{pmatrix} m_1 \\ m_2 \\ \tilde{m}_2 \\ -\tilde{m}_1 \\ q_i \end{pmatrix} , \ \begin{pmatrix} \tilde{m}_1 \\ \tilde{m}_2 \\ \tilde{n}_1 \\ \tilde{n}_2 \\ \tilde{q}_i \end{pmatrix} \rightarrow \begin{pmatrix} -n_2 \\ n_1 \\ \tilde{n}_1 \\ \tilde{n}_2 \\ \tilde{q}_i \end{pmatrix} $$

• If the shift acts on $(m_1, m_2)$ in heterotic, it remains perturbative in type II. The gravitino masses are functions of the geometric moduli.

• If the shift acts on the (0,16) lattice in heterotic, it affects non-perturbative, D-brane related (electric) states in type II. The gravitino masses now also involve the Wilson lines that in type-II are RR fields.

• If the shift is along windings $(n_1, n_2)$, then it affects only non-perturbative magnetic states in type II. For example a shift in $n_2$ gives a gravitino mass
in type two of the form

$$M_{\text{BPS}}^2 = \frac{|S|^2}{4T_2(S_U^2 - \frac{1}{2} \sum_i \text{Im}W_i^2)}$$

in units of the Planck scale. The gravitino is a NS5-related soliton.

- In perturbation theory this is similar to a hard breaking $\mathcal{N} = 4 \rightarrow \mathcal{N} = 2$.

- In global supersymmetry, this has been described by using magnetic Fayet-Iliopoulos terms.

Antoniadis+Partouche+Taylor-95
A general non-perturbative $\mathcal{N} = 4 \rightarrow \mathcal{N} = 2$ breaking

- We will choose a $\mathbb{Z}_2$ action for concreteness.
- Choose an electric and a magnetic $\mathbb{Z}_2$ SS shift vector $(\epsilon, \bar{\epsilon})$.
- Satisfying a generalization of modular invariance condition Orbit of $(\epsilon^2, \bar{\epsilon}^2, 2\epsilon \cdot \bar{\epsilon}) = (1, 0, 0) \mod 4$.
- States satisfying $q_e \cdot \epsilon + q_m \cdot \bar{\epsilon} = 1 \mod 2$ with lowest mass are the massive gravitini.
- In the even sector ($h = \bar{h} = 0$), states with $q_e \cdot \epsilon + q_m \cdot \bar{\epsilon} = \text{even/odd}$ are hyper/vector multiplets.
- In sectors with $h$ or $\bar{h} \neq 0$, the $q_e, q_m$ are shifted appropriately by half-integers.

BPS multiplets ...,
Non-perturbative BPS multiplicities

The strategy:

(a) Calculate the perturbative multiplicities using the helicity supertraces

(b) extend them to magnetic dyons, using the DVV arguments

•

\[ F_1 = \frac{\bar{\vartheta}_3 \bar{\vartheta}_4}{\bar{\eta}^{24}}, \quad F_\pm = \frac{\bar{\vartheta}_2(\bar{\vartheta}_3 \pm \bar{\vartheta}_4)}{\bar{\eta}^{24}} \]

associated with the sublattices

\[ F_1 \leftrightarrow \frac{\Gamma_{2,18}^0[0] + \Gamma_{2,18}^0[0]}{2} \quad (\text{vectors or hypers}) \]

\[ F_\pm \leftrightarrow \frac{\Gamma_{2,18}^1[0] + \Gamma_{2,18}^1[0]}{2} \quad (\text{hypers}) \]
The derive the non-perturbative extension we first rewrite the perturbative generating functions as

\[
F_1 = \frac{1}{\eta^{24}} \chi \left[ \begin{array}{c} 0 \\ 1 \end{array} \right], \quad F_\pm = \frac{1}{\eta^{24}} \left( \bar{\chi} \left[ \begin{array}{c} 1 \\ 0 \end{array} \right] \pm \bar{\chi} \left[ \begin{array}{c} 1 \\ 1 \end{array} \right] \right)
\]

\[
\bar{\chi} \left[ \begin{array}{c} h \\ g \end{array} \right] = \frac{1}{8} \frac{1}{\eta^6} \sum_{a,b} (-)^h \bar{\vartheta}^4[b+g] \bar{\vartheta}^4[b-g] \bar{\vartheta}^{1+h}[1+g] \bar{\vartheta}^{1-h}[1-g].
\]

At genus-2 \( h \) and \( g \) become \( \vec{h} = (h, \tilde{h}) \) and \( \vec{g} = (g, \tilde{g}) \) in correspondence with the “electric” and “magnetic” charge shifts.

\[
\bar{\vartheta}^4[a+h][b+g](\bar{\tau}) \rightarrow \bar{\vartheta}^4[a+\vec{h}][b+\vec{g}](\vec{\tau}^{ij}).
\]

Then, the non-perturbative multiplicities are generated by

\[
F \left[ \begin{array}{c} \vec{h} \\ \vec{g} \end{array} \right] = \Phi(\vec{\tau}^{ij}) \bar{\chi} \left[ \begin{array}{c} \vec{h} \\ \vec{g} \end{array} \right](\vec{\tau}^{ij}),
\]

where \( \Phi(\vec{\tau}^{ij}) \) is the \( N = 4 \) multiplicity function.

The expansion of the genus-2 result is similar to the \( \mathcal{N} = 4 \) case.
String thresholds and SS supersymmetry breaking

- String threshold corrections in vacua with spontaneously broken susy, have rather different properties.

- The calculations can be done either by integrating one-loop amplitudes
  \[ \text{Kaplunovsky-87, Dixon+Kaplunovsky+Louis-91} \]
  or using the background field formalism
  \[ \text{Kounnas+Kiritsis-94, Petropoulos+Rizos-96} \]
  \[ \text{Kiritsis+Kounnas+Petropoulos+Rizos-96, Bachas+Fabre-96} \]

- Unlike the linear behavior in the decompactification volume, a logarithmic behavior is observed
  \[ \lim_{T_2 \to \infty} \frac{1}{g^2} \sim \log T_2 + \mathcal{O}(e^{-T_2}) \]
  \[ \text{Kiritsis+Kounnas+Petropoulos+Rizos-96-98} \]

- There are similar universality $\mathcal{N} = 2$ properties as in the case of usual orbifolds.

- The non-perturbative prepotential in the $\mathcal{N} = 2$ case can be calculated by duality techniques. No known technique for general non-perturbative SS breaking.

BPS multiplets \,..., \hspace{1cm} \text{E. Kiritsis} \hspace{1cm} 21
• The qualitative features of supersymmetry breaking by gaugino condensation in the heterotic string are similar to SS supersymmetry breaking along the eleventh dimension  

  \textit{Antoniadis+Quiros-97}

• This led to SS supersymmetry breaking on orientifolds  

  \textit{Antoniadis+Dudas+Sagnotti-98}  
  \textit{Antoniadis+D'Appollonio+Dudas+Sagnotti-98}  
  \textit{Angelantonj+Antoniadis+Förger-99}

Two options:

♠ Translations parallel to branes break susy in the open sector

♠ Translation transverse to branes leaves massless brane spectrum supersymmetric.

• Alternative sources of susy breaking: internal magnetic fields on D-branes  

  \textit{Bachas-96}  
  \textit{Angelantonj+Antoniadis+Dudas+Sagnotti-00}  
  \textit{Blumenhagen+Goerlich+Kors+Lüst-00}

BPS multiplets ...
Epilogue

- Spontaneous supersymmetry breaking in string theory followed the cue of Scherk and Schwarz in 1979
  
  - Its original formulation was tied to the idea of an internal flux
  
  - It was later realized it is equivalent to freely acting orbifolds.
  
  - In conjunction with BPS techniques, it gave insights into the non-perturbative structure of spontaneously broken extended supergravity theories.

- It is one of the two most popular mechanisms of breaking susy in string theory.
  
  - It is one of the successful examples of the interplay between string theory and supergravity.
Detailed plan of the presentation

- **Title page**: 0 minutes
- **Plan**: 1 minute
- **Ouverture**: 3 minutes
- **The supersymmetry algebra and BPS states**: 7 minutes
- **BPS states and non-perturbative physics**: 9 minutes
- **Helicity supertraces**: 12 minutes
- **The BPS mass formula: the $\mathcal{N}=4$ paradigm**: 19 minutes
- **An $\mathcal{N}=2$ truncation**: 22 minutes
- **Heterotic/type-II duality and branes**: 24 minutes
- **Matching the type-II BPS spectrum**: 25 minutes
- **The magnetic spectrum and the duality**: 27 minutes
- **Spontaneous supersymmetry breaking (à la Scherk-Schwarz)**: 33 minutes
- **Spontaneous versus hard supersymmetry breaking**: 35 minutes
- **Generic partial spontaneous susy breaking**: 37 minutes
• (Internal) Fluxes and supersymmetry breaking (the prehistory) 41 minutes
• Duality and non-perturbative supersymmetry breaking 45 minutes
• A general non-perturbative $\mathcal{N} = 4 \rightarrow \mathcal{N} = 2$ breaking 49 minutes
• Non-perturbative BPS multiplicities 51 minutes
• String threshold correction and SS supersymmetry breaking 52 minutes
• Supersymmetry breaking in orientifolds 53 minutes
• Epilogue 55 minutes