30 years of Supergravity

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BPS multiplets, supersymmetry breaking and non-perturbative effects

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Prologue

• I have been one of the users of supergravity in my scientific life, but I have never inhaled.

• I have never met J. Scherk, but I have worked on two (of the many) aspects of his mark in theoretical physics:

♠ Anti-gravity, aka BPS states.

♠ Geometrical Supersymmetry breaking (as originally conceived by J. Scherk and J. Schwarz).

The Plan

- Ouverture

- The supersymmetry algebra and BPS states

- BPS saturated couplings and helicity supertraces

- BPS mass formlulae:the $\mathcal{N} = 4$ paradigm.

- Scherk-Schwarz supersymmetry breaking in perturbation theory

- Duality and Scherk-Schwarz supersymmetry breaking beyond perturbation theory

- Epilogue

Ouverture

• Supersymmetry has spread its magic over the last 40 decades

• It quickly met, head on, gravity to form supergravity and raise the stakes.

• Supergravity quickly teamed up with string theory in an unparalleled match to dominate theoretical attempts at unification in the last two decades.

• The Supergravity/String theory match was turbulent at first. String theorists looked with suspicion or disdain at supergravity. Supergravity theorists felt for a while out of place and time.

• It is supersymmetry and the need to understand strong coupling limits that provided a meeting point, and since the mid-nineties the two directions go hand-in-hand.

• It is the meeting point, via BPS multiplets, supersymmetry and its breaking, and non-perturbative effects, that I will review here. The review will be short, and dominated by the things I understand best.

The supersymmetry algebra and BPS states

• The key to BPS multiplets and their magic lies in the special properties of the supersymmetry algebra.

$$\{Q^I_\alpha, Q^J_\beta\} = \epsilon_{\alpha\beta} Z^{IJ} \quad , \quad \{\bar{Q}^I_{\dot\alpha}, Q^J_{\dot\beta}\} = \epsilon_{\dot\alpha\dot\beta} \bar{Z}^{IJ} \quad , \quad \{Q^I_\alpha, \bar{Q}^J_{\dot\alpha}\} = \delta^{IJ}\ 2\sigma^\mu_{\alpha\dot\alpha} P_\mu$$

$I = 1, 2, \cdots, \mathcal{N}$.

• Invariance under the U($\mathcal{N}$) R-symmetry that rotates $Q, \bar{Q}$.

• Massive representations. Go to the rest frame $P \sim (M, \vec{0})$.

$$\{Q^I_\alpha, \bar{Q}^J_{\dot\alpha}\} = 2M\delta_{\alpha\dot\alpha}\delta^{IJ} \quad , \quad \{Q^I_\alpha, Q^J_\beta\} = \{\bar{Q}^I_{\dot\alpha}, \bar{Q}^J_{\dot\beta}\} = 0$$

• Define the $2\mathcal{N}$ fermionic harmonic creation and annihilation operators

$$A^I_\alpha = \frac{1}{\sqrt{2M}} Q^I_\alpha \quad , \quad A^{\dagger I}_\alpha = \frac{1}{\sqrt{2M}} \bar{Q}^I_{\dot\alpha}$$

• Start with the Clifford vacuum $|\Omega\rangle$, (annihilated by the A^I_α) and act with $A^{\dagger I}_\alpha$.

$$\text{total number of states} \quad = \quad \sum_{n=0}^{2\mathcal{N}} \binom{2\mathcal{N}}{n} = 2^{2\mathcal{N}}$$

• Massless representations. Go to the frame $P \sim (-E, 0, 0, E)$

$$\{Q^I_\alpha, \bar{Q}^J_{\dot\alpha}\} = 2 \begin{pmatrix} 2E & 0 \\ 0 & 0 \end{pmatrix} \delta^{IJ}$$

$Q^I_2, \bar{Q}^I_{\dot{2}} = 0$ for such reps. The representation is $2^{\mathcal{N}}$-dimensional. This is a trivial example of a BPS rep.

• Non-zero central charges. Skew-diagonalize Z^{IJ} with real positive skew-eigenvalues Z_m.

$$\{Q^{am}_\alpha, \bar{Q}^{bn}_{\dot{\alpha}}\} = 2M\delta^{\alpha\dot{\alpha}}\delta^{ab}\delta^{mn} \quad , \quad \{Q^{am}_\alpha, Q^{bn}_\beta\} = Z_n\epsilon^{\alpha\beta}\epsilon^{ab}\delta^{mn}$$

• Define

$$A^m_\alpha = \frac{1}{\sqrt{2}}[Q^{1m}_\alpha + \epsilon_{\alpha\beta}Q^{2m}_\beta] \quad , \quad B^m_\alpha = \frac{1}{\sqrt{2}}[Q^{1m}_\alpha - \epsilon_{\alpha\beta}Q^{2m}_\beta]$$

$$\{A^m_\alpha, A^{\dagger n}_\beta\} = \delta_{\alpha\beta}\delta^{mn}(2M + Z_n) \quad , \quad \{B^m_\alpha, B^{\dagger n}_\beta\} = \delta_{\alpha\beta}\delta^{mn}(2M - Z_n)$$

• From Unitarity → Bogomolnyi bound

$$M \geq \max\left[\frac{Z_m}{2}\right]$$

• Assume $0 \leq r \leq \mathcal{N}/2$ of the $Z_m = 2M$. Then $2r$ of the B-oscillators vanish identically. We are left with $2\mathcal{N} - 2r$ creation and annihilation operators. The representation has $2^{2\mathcal{N}-2r}$ states. The maximal case, has as many states as the massless multiplet.

BPS states and non-perturbative physics

• Central charges depend on couplings and vevs.

• Massive BPS states can become massless without interference from other multiplets. The inverse is also true.

• Massive BPS states are absolutely stable in large regions of the moduli space. They can be reliably extrapolated at strong coupling

• There are special effective field theory couplings that obtain quantum corrections only from BPS states. They are termed “BPS-saturated couplings”. They have special properties, but they typically include the lowest energy relevant couplings

• Helicity supertrace formulae are at the heart of the connection between BPS states and quantum corrections to effective couplings.

• All successes of duality conjectures and non-perturbative determinations of EFTs à la Seiberg-Witten rely on BPS-saturated couplings.

Helicity supertraces

• The role of helicity supertraces in β-function calculations and in issues of supersymmetry breaking is known since 1981
Curtright-81, Ferrara+Savoy+Girardello-81

• It was realized that they are central in quantitative tests of duality conjectures
Bachas+Kiritsis-96
Gregori+Kiritsis+Kounnas+Obers+Petropoulos+Pioline-97
Bachas+Fabre+Kiritsis+Obers+Vanhove-97
Kiritsis+Obers-97
Gregori+Kounnas+Petropoulos-98

• In 4d they are defined as

$$B_{2n}(R) = \text{Tr}_R[(-1)^{2\lambda}\lambda^{2n}]$$

• The "helicity-generating function" of a given supermultiplet R

$$Z_R(y) = \text{Str}\ y^{2\lambda}\,.$$

• For a particle of spin j we have

$$Z_{[j]} = (-)^{2j}\Big(\frac{y^{2j+1}-y^{-2j-1}}{y-y^{-1}}\Big)\ \ \text{massive}\quad ,\quad Z_{[}j] = (-)^{2j}(y^{2j}+y^{-2j})\quad \text{massless}$$

• The supertrace of the n-th power of helicity can be extracted from the generating functional through

$$B_n(R) = (y^2\frac{d}{dy^2})^n\ Z_R(y)|_{y=1}$$

• An $\mathcal{N}=2$ example:This is relevant for the two derivative N=2 effective action

$$B_0(\text{any}) = 0\quad ,\quad B_2(M_\lambda) = (-1)^{2\lambda+1}\quad ,\quad B_0(S_j) = (-1)^{2j+1}j(j+1)\quad ,\quad B_2(M_j) = 0$$

• For $\mathcal{N}=4$ $B_0 = B_2 = 0,\ B_4(L) = B_6(L) = 0,\ B_4(I) = 0$

The BPS mass formula:the $\mathcal{N}$=4 paradigm

• We consider $\mathcal{N}$=4 supergravity coupled to n vector multiplets. We take n=22, relevant to the heterotic/T^6 compactifications or type II/K3$\times T^2$

• The scalar space is SU(1,1)/U(1)$\times$ O(6,22)/(O(6)$\times$O(22)). The first part is parameterized by the complex S field while the second by an O(6,22) symmetric matrix M,

$$M^T = M \quad , \quad M^T L M = L \quad , \quad L = \begin{pmatrix} 0 & \mathbf{1}_6 & 0 \\ \mathbf{1}_6 & 0 & 0 \\ 0 & 0 & -\mathbf{1}_{16} \end{pmatrix}$$

In the heterotic theory it is a function of the internal components of G, B, A^I

$$S = S_1 + iS_2 = a + i\, e^{-2\phi}$$

• The gauge and scalar action is

$$\mathcal{L} \sim \left[-4\frac{\partial^\mu S \partial_\mu \bar{S}}{ImS^2} - 2ImS(M^{-1})_{ij}F^i_{\mu\nu}F^{j,\mu\nu} + 2ReS\ L_{ij}F^i \wedge F^j + \mathrm{Tr}(\partial_\mu M \partial^\mu M^{-1}) \right]$$

There is a perturbative invariance under O(6,22,Z) transformations

$$M \to \Omega\, M\Omega^T \quad , \quad F_{\mu\nu} \to \Omega\, F_{\mu\nu} \quad , \quad \Omega \in O(6,22,Z)$$

with electric charges transforming as

$$e_i \to \Omega_{ij} e_j$$

- Finally there is electric magnetic duality

$$S \to \frac{aS+b}{cS+d} \quad , \quad M \to M \quad , \quad F^i_{\mu\nu} \to (c\,\mathrm{Re}S + d)F^i_{\mu\nu} + c\,\mathrm{Im}S\;(ML)_{ij}\;{}^*F^j_{\mu\nu}$$

- We parametrize the electric and magnetic charges of generic dyons

$$\vec{Q}_e = \frac{1}{\sqrt{2}\,\mathrm{Im}S} M(\vec{\alpha} + \mathrm{Re}S\;\vec{\beta}) \quad , \quad \vec{Q}_m = \frac{1}{\sqrt{2}} L\,\vec{\beta} \quad , \quad \vec{\alpha}, \vec{\beta} \in \mathbb{Z}^{28}$$

so that they satisfy the Dirac-Schwinger-Zwanziger-Witten quantization condition.

Electric-Magnetic duality

- The BPS mass formula can be expressed in two equivalent ways

Cvetic+Youm-95, Cvetic+Tseytlin-95

$$M^2_{BPS} = \frac{ImS}{4}\left[Q_e^t\tilde{M}_+Q_e + Q_m^t\tilde{M}_+Q_m + 2\sqrt{(Q_e^t\tilde{M}_+Q_e)(Q_m^t\tilde{M}_+Q_m) - (Q_e^t\tilde{M}_+Q_m)^2}\,\right]$$

$$= \frac{1}{4ImS}(\alpha^t + S\beta^t)M_+(\alpha + \bar{S}\beta) + \frac{1}{2}\sqrt{(\alpha^t M_+\alpha)(\beta^t M_+\beta) - (\alpha^t M_+\beta)^2}$$

with $M_+ = M + L$ and $\tilde{M}_+ = LM_+L$.

- It is O(6,22,$\mathbb{Z}$) invariant

$$\alpha \rightarrow \Omega\,\alpha \quad , \quad \beta \rightarrow \Omega\,\beta \ ,$$

and SL(2,$\mathbb{Z}$)$_S$ invariant

$$\begin{pmatrix} \vec{\alpha} \\ \vec{\beta} \end{pmatrix} \rightarrow \begin{pmatrix} a & -b \\ -c & d \end{pmatrix} \begin{pmatrix} \vec{\alpha} \\ \vec{\beta} \end{pmatrix}$$

- The square-root factor is proportional to the difference of the two central charges squared.

- It is zero when $\vec{\beta} \sim \alpha$ and the we have 1/2 BPS multiplets. Otherwise we have 1/4 BPS multiplets.

- For perturbative BPS states of the heterotic string, $\vec{\beta} = 0$.

$$M^2_{BPS,\text{pert}} = \frac{1}{4\ ImS}\,\alpha^t \cdot M_+ \cdot \alpha = \frac{1}{4\ ImS}\,p_L^2$$

- The multiplicity of 1/2 perturbative heterotic BPS states is given by the B_4 helicity formula:

$$d_N \quad states \quad with \quad N = \frac{1}{2}\alpha^t \cdot L \cdot \alpha \quad , \quad \frac{1}{\eta^{24}} = \sum_{N=-1}^{\infty} d_N\ q^N$$

- The non-perturbative multiplicity is conjectured to be

Dijkgraaf+Verlinde+Verlinde-96

$$d_{N_e,N_m,N_s} \quad states \quad with \quad N_e = \frac{1}{2}\alpha^t \cdot L \cdot \alpha \quad , \quad N_m = \frac{1}{2}\beta^t \cdot L \cdot \beta \quad , \quad N_s = \alpha^t \cdot L \cdot \beta$$

$$\frac{1}{\Phi(\Omega)} = \sum_{N_e,N_m,N_s} d_{N_e,N_m,N_s}\; e^{-2\pi i(N_e\rho+N_m\tau+N_s v)} \quad , \quad \Omega = \begin{pmatrix} \rho & v \\ v & \tau \end{pmatrix}$$

• It correctly accounts for the associated black-hole entropy of dyonic branes (1/4 BPS).

$$S = \pi\sqrt{(\alpha^t \cdot L \cdot \alpha)(\beta^t \cdot L \cdot \beta) - (\alpha^t \cdot L \cdot \beta)^2}$$

• Recently, even the 1/2 BPS black-holes have been accounted by the perturbative multiplicity formula.

Dabholkar-04,Sen-05, Dabholkar+Denef+Moore+Pioline-05

An N=2 truncation

• We may freeze the (4,22) moduli, and focus on the remaining (2,2+16) associated with a single T^2.

• This is also an $\mathcal{N}$=2 truncation, appropriate for K3×T^2 compactifications.

• The T, U and W^i moduli are defined as

$$G = \frac{T_2 - \frac{1}{2U_2}\sum_i (\text{Im}W_i)^2}{U_2} \begin{pmatrix} 1 & U_1 \\ U_1 & |U|^2 \end{pmatrix} \ , \ B = \left(T_1 - \frac{\sum_i \text{Re}W_i \text{Im}W_i}{2U_2} \right) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$W_i = -Y_2^i + UY_1^i \quad , \quad K = -log[T_2 U_2 - \frac{1}{2}\sum_i \text{Im}W_i^2]$$

• The BPS mass formula now is

Kiritsis+Kounnas-97

$$M^2_{BPS} = \frac{\Big| -m_1 U + m_2 + T n_1 + (TU - \frac{1}{2}\sum_i W_i^2) n_2 + W_i q^i +}{4\, S_2 \left(T_2 U_2 - \frac{1}{2}\sum_i \mathrm{Im} W_i^2\right)}$$

$$+ \frac{S[-\tilde m_1 U + \tilde m_2 + T \tilde n_1 + \tilde n_2 (TU - \frac{1}{2}\sum_i W_i^2) + \tilde q^i W_i]\Big|^2}{4\, S_2 \left(T_2 U_2 - \frac{1}{2}\sum_i \mathrm{Im} W_i^2\right)}$$

• Invariant under $SL(2,\mathbb{Z}) \times O(2,18,\mathbb{Z})$

• O(2,18,$\mathbb{Z}$) acts in the standard fashion. SL(2,$\mathbb{Z}$) acts by interchanging electric and magnetic charges.

$$S \to -\frac{1}{S} \quad , \quad \begin{pmatrix} m_i \\ n_i \\ q_i \\ \tilde m_i \\ \tilde n_i \\ \tilde q_i \end{pmatrix} \to \begin{pmatrix} \tilde m_i \\ \tilde n_i \\ \tilde q_i \\ -m_i \\ -n_i \\ -q_i \end{pmatrix} \quad , \quad S \to S+1 \quad , \quad \begin{pmatrix} m_i \\ n_i \\ q_i \\ \tilde m_i \\ \tilde n_i \\ \tilde q_i \end{pmatrix} \to \begin{pmatrix} m_i - \tilde m_i \\ n_i - \tilde n_i \\ q_i - \tilde q_i \\ \tilde m_i \\ \tilde n_i \\ \tilde q_i \end{pmatrix} \quad ,$$

Heterotic/type-II duality and branes

• heterotic/T^6 is non-perturbatively equivalent to type II/K3$\times T^2$.

The simple tests:

• The non-perturbative states (monopoles) on heterotic side are fundamental particles on type II and vice versa. The heterotic NS5-brane on T^4 is dual to the perturbative type IIA string and vice versa. *Hull+Townsend-94, Witten-95, M. Duff-95*

• The charged states $q^i \neq 0$ are RR states in the type II side. The non-abelian completion comes from the type-II D-branes. *Hull+Townsend-94, Witten-95*

• BPS multiplicities agree. *Bershadsky+Sadov+Vafa-96*

• The $\mathcal{F}^4$ thresholds on the two side quantitatively agree (One-loop $\leftrightarrow$ tree level). *Kiritsis+Obers+Pioline-00*

• Indirect tests: upon reduction of susy: heterotic on K3$\times T^2$ $\sim$ type-II on CY (elliptic K3 fibration). *Kachru+Vafa-95, Ferrara+Harvey+Strominger+Vafa-95*

Matching the type-II BPS spectrum

The key: emergence of massless vectors

• C_3 and C_1 on K3, give rise to (4,20) vectors, matching the Hodge diamond of K3.

• The charged states are: D4 branes wrapped on K3, D0 branes and D2 branes wrapped on the (3,19) cycles dual to the two-forms. This generates the $\Gamma_{4,20}$ lattice where the integers are wrapping and multiplicity numbers for the D-branes.

Upon descent to 4 dimensions we have $\Gamma_{2,2} \times \Gamma_{4,20} \to \Gamma_{6,22}$ There for (2,2) from the vectors are NS-NS while the rest (4,20) are RR.

The magnetic spectrum and the duality

• On the heterotic side, the magnetic states arise from the NS5-brane. Wrapped around T^6 it gives rise to a magnetic copy of the electric spectrum. The associated masses scale according to the BPS formula as

$$|S|^2/S_2 \sim \frac{1}{g_s^2}$$

• On the type-II side, the missing states are also generated by the NS5 branes wrapped around K3$\times T^2$.

• Heterotic/type II duality involves

$$S \leftrightarrow T$$

and a dualization of the NS two-form that translates into:

$$T \leftrightarrow S \quad , \quad \begin{pmatrix} m_1 \\ m_2 \\ n_1 \\ n_2 \\ q_i \end{pmatrix} \to \begin{pmatrix} m_1 \\ m_2 \\ \tilde{m}_2 \\ -\tilde{m}_1 \\ q_i \end{pmatrix} \quad , \quad \begin{pmatrix} \tilde{m}_1 \\ \tilde{m}_2 \\ \tilde{n}_1 \\ \tilde{n}_2 \\ \tilde{q}_i \end{pmatrix} \to \begin{pmatrix} -n_2 \\ n_1 \\ \tilde{n}_1 \\ \tilde{n}_2 \\ \tilde{q}_i \end{pmatrix}$$

• These transformations are valid, even if we include the (4,4) part of the lattice.

Spontaneous supersymmetry breaking (à la Scherk-Schwarz)

• The Scherk-Schwarz Susy breaking has been successfully implemented in string theory

Rohm-84, Kounnas+Porrati-88,Ferarra+Kounnas+Porrati+Zwirner-89, Kounnas+Rostand-90

• It has been shown to be generated by appropriate freely-acting orbifolds.

Kiritsis+Kounnas-97

• We will study here first the $\mathcal{N} = 4 \to \mathcal{N} = 2$ partial breaking in the toroidal heterotic string. We will then use BPS data and duality to construct non-perturbative models of susy breaking.

Kiritsis+Kounnas+Petropoulos+Rizos-96-98

• To start, we choose moduli to factorize the lattice

$$\Gamma_{6,22} \quad \to \quad \Gamma_{2,18} \quad \times \quad \Gamma_{4,4}$$

• We consider an orbifold that acts as a $\mathbb{Z}_N$ rotation on $\Gamma_{4,4}$ and by a $\mathbb{Z}_N$ translation ϵ/N, with $\epsilon \equiv (\epsilon_L, \epsilon_R, \zeta)$ on $\Gamma_{2,18}$. The rotation breaks half of the supersymmetry

• Modular invariance implies

$$\frac{\epsilon^2}{2} \equiv \epsilon_L \cdot \epsilon_R - \frac{1}{2}\zeta \cdot \zeta = 1 \text{ mod } N^2$$

• T-duality acts both on moduli (T, U, W_i) and the shift vector ϵ. ϵ^2 is T-duality invariant.

• The mass of the two gravitini:

$$M^2_{\text{gravitino}} = \frac{\left| -m_1 U + m_2 + T n_1 + (TU - \frac{1}{2}\sum_i W_i^2) n_2 + W_i q^i \right|^2}{4\ S_2 \left(T_2 U_2 - \frac{1}{2}\sum_i \text{Im} W_i^2 \right)}$$

$$m \cdot \epsilon_R + n \cdot \epsilon_L - q \cdot \zeta = \pm 1 \text{ mod } N$$

• Many solutions , but the two lightest ones are the massive gravitini. The rest are KK descendants. This depend on the region of moduli space.

• The massive gravitons are BPS states of the unbroken $\mathcal{N}$=2 supersymmetry.

• The full BPS spectrum (half-BPS vector multiplets and hypermultiplets can be easily computed via the B_2 helicity supertrace.

It is given by the O(2,18) BPS mass formula. In the "untwisted" sector

$$M^2_{\mathrm{BPS}} = \frac{\left| -m_1 U + m_2 + T n_1 + (TU - \frac{1}{2}\sum_i W_i^2) n_2 + W_i q^i \right|^2}{4\; S_2 \left(T_2 U_2 - \frac{1}{2}\sum_i \mathrm{Im} W_i^2\right)}$$

They are hypermultiplets when

$$m \cdot \epsilon_R + n \cdot \epsilon_L - q \cdot \zeta = \quad even$$

and vector-like multiplets when

$$m \cdot \epsilon_R + n \cdot \epsilon_L - q \cdot \zeta = \ odd$$

In the twisted sector

$$(m, n, q) \to \left(m + \frac{\epsilon_L}{2}, n + \frac{\epsilon_R}{2}, q + \frac{\zeta}{2} \right)$$

Spontaneous versus hard supersymmetry breaking

• We would like to compare spontaneous SUSY breaking à la SS and "hard" susy breaking.

• Consider a Z_2 orbifold action that breaks susy. All massless gravitini are removed from the spectrum. The spectrum up to M_s is non-supersymmetric.

• Compare with a similar orbifold where the Z_2 action is accompanied by a Z_2 shift on a circle of radius R. Here the gravitini have masses $\sim 1/R$

♠When $R \to \infty$ supersymmetry is restored. $R > 1$ is the softly broken region.

♣When $R \to 0$, the massive gravitini decouple and the vacuum is identical to the hard susy-breaking one (provided susy is not completely broken)

Generic partial spontaneous susy breaking

• By appropriate non-free action we may break susy in various other ways:

♠ $\mathcal{N}=4 \to \mathcal{N}=1$

♠ $\mathcal{N}=4 \to \mathcal{N}=0$

By accompanying these actions with lattice shifts we obtain vacua with various type of partial supersymmetry breaking.

Simple illustrative Examples:

• $\mathcal{N}=4 \to \mathcal{N}=0$: Heterotic string in finite temperature: $(-1)^F$ plus Z_2 translation along Euclidean time.

Rohm-84, Attick+Witten-88
Kounnas+Ronstant-90
Antoniadis+Derendiger+Kounnas-98

• $\mathcal{N}=4 \to \mathcal{N}=1$: $\mathbb{Z}_2 \times \mathbb{Z}_2$ action on $T^2 \times T^2 \times T^2$ with a translation in the transverse torus.

Kiritsis+Kounnas-97, Gregori+Kounnas+Rizos-99

(Internal) Fluxes and supersymmetry breaking (the prehistory)

• It was realized early on this supersymmetry breaking could be interpreted as due to special discrete internal fluxes

Kounnas-87, Kiritsis+Kounnas-97

• The (discrete) perturbations have the form

$$\Delta S \sim \int d^2z \;\; F^a_{ij} \;\; [\psi^i\psi^j - X^{(i}\partial X^{j)}] \;\; \bar{J}^a$$

They are constrained (quantized) by requiring preservation of world-sheet susy

• The appropriate discrete operator $U = e^{\oint dz \;\; F^a_{ij} \;\; [\psi^i\psi^j - X^{(i}\partial X^{j)}]}$ can be constructed by fermionization of X^I.

• They generate non-vanishing auxiliary fields in 4d.

• For example the $\mathcal{N}=4 \to \mathcal{N}=2$ breaking is due to a self-dual "flux", $F_{34} = -F_{55}$.

• Their type-II duals correspond to special points of flux compactifications with both RR and NS fluxes turned on.

Kiritsis+Kounnas-97

Since then, several advances:

• Superpotentials have been advanced to deal with fluxes

Taylor+Vafa-99, Gukov+Witten+Vafa-01

• The effects of non-trivial fluxes in the type II context was resurrected with different motivation (warping) several years later

Giddings+Kachru+Polchinski-01

• Supersymmetry breaking was rediscussed in this framework, together with the important issue of moduli stabilisation.

Kachru+Schultz+Trivedi-02
Kachru+Kallosh+Linde+Trivedi-03

Duality and non-perturbative supersymmetry breaking

• We consider for simplicity a heterotic Z_2 freely-acting orbifold with a shift vector ϵ acting on $\Gamma_{2,18}$.

• According to the "adiabatic argument" a similar action on the type II side will give a dual ground-state.

Vafa+Witten-95

The Z_2 rotation acts as $(-1)^{F_L}$ accompanied by an involution e that changes the sign of 16 of the self-dual two-forms on K3.

• The $\Gamma_{2,18}$ (electric) shift vector $\epsilon = (\epsilon_L, \epsilon_R, \zeta)$ gives Z_2 phases to various states:

• The (0,16) shift can be associated with (discrete) RR fluxes threading the appropriate cycles of K3.

• To see further the correspondence, I recall the heterotic/type-II map on T^2 charges:

$$T \leftrightarrow S \quad , \quad \begin{pmatrix} m_1 \\ m_2 \\ n_1 \\ n_2 \\ q_i \end{pmatrix} \rightarrow \begin{pmatrix} m_1 \\ m_2 \\ \tilde{m}_2 \\ -\tilde{m}_1 \\ q_i \end{pmatrix} \quad , \quad \begin{pmatrix} \tilde{m}_1 \\ \tilde{m}_2 \\ \tilde{n}_1 \\ \tilde{n}_2 \\ \tilde{q}_i \end{pmatrix} \rightarrow \begin{pmatrix} -n_2 \\ n_1 \\ \tilde{n}_1 \\ \tilde{n}_2 \\ \tilde{q}_i \end{pmatrix}$$

• If the shift acts on (m_1, m_2) in heterotic, it remains perturbative in type II. The gravitino masses are functions of the geometric moduli.

• If the shift acts on the (0,16) lattice in heterotic, it affects non-perturbative, D-brane related (electric) states in type II. The gravitino masses now also involve the Wilson lines that in type-II are RR fields.

• If the shift is along windings (n_1, n_2), then it affects only non-perturbative magnetic states in type II. For example a shift in n_2 gives a gravitino mass in type two of the form

$$M^2_{\text{BPS}} = \frac{\Big|S\Big|^2}{4\,T_2\left(S_2U_2 - \frac{1}{2}\sum_i \text{Im}W_i^2\right)}$$

in units of the Planck scale. The gravitino is a NS5-related soliton.

• In perturbation theory this is similar to a hard breaking $\mathcal{N}=4 \to \mathcal{N}=2$.

• In global supersymmetry, this has been described by using magnetic Fayet-Iliopoulos terms.

Antoniadis+Partouche+Taylor-95

A general non-perturbative $\mathcal{N}=4 \to \mathcal{N}=2$ breaking

- We will choose a $\mathbb{Z}_2$ action for concreteness.

- Choose an electric and a magnetic $\mathbb{Z}_2$ SS shift vector $(\epsilon, \tilde{\epsilon})$.

- Satisfying a generalization of modular invariance condition
Orbit of $(\epsilon^2, \tilde{\epsilon}^2, 2\epsilon \cdot \tilde{\epsilon}) = (1,0,0)\ mod\ 4$.

- States satisfying $q_e \cdot \epsilon + q_m \cdot \tilde{\epsilon} = 1 \ \ mod \ \ 2$ with lowest mass are the massive gravitini.

- In the even sector ($h = \tilde{h} = 0$), states with $q_e \cdot \epsilon + q_m \cdot \tilde{\epsilon} = even/odd$ are hyper/vector multiplets

- In sectors with h or $\tilde{h} \neq 0$, the q_e, q_m are shifted appropriately by half-integers.

Non-perturbative BPS multiplicities

The strategy:

(a) Calculate the perturbative multiplicities using the helicity supertraces

(b) extend them to magnetic dyons, using the DVV arguments

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$$\bar{F}_1 = \frac{\bar{\vartheta}_3^2\bar{\vartheta}_4^2}{\bar{\eta}^{24}} \quad , \quad \bar{F}_\pm = \frac{\bar{\vartheta}_2^2(\bar{\vartheta}_3^2 \pm \bar{\vartheta}_4^2)}{\bar{\eta}^{24}}$$

associated with the sublattices

$$F_1 \leftrightarrow \frac{\Gamma_{2,18}[^0_0] \pm \Gamma_{2,18}[^0_1]}{2} \qquad (vectors \quad or \quad hypers)$$

$$F_\pm \leftrightarrow \frac{\Gamma_{2,18}[^1_0] \pm \Gamma_{2,18}[^1_1]}{2} \qquad (hypers)$$

• The derive the non-perturbative extension we first rewrite the perturbative generating functions as

$$F_1 = \frac{1}{\bar\eta^{24}}\ \chi\begin{bmatrix}0\\1\end{bmatrix}, \quad F_\pm = \frac{1}{\bar\eta^{24}}\ \left(\bar\chi\begin{bmatrix}1\\0\end{bmatrix} \pm \bar\chi\begin{bmatrix}1\\1\end{bmatrix}\right)$$

$$\bar\chi\begin{bmatrix}h\\g\end{bmatrix} \ = \ \frac{1}{8\ \bar\eta^6}\ \sum_{a,b}(-)^h\ \bar\vartheta^4[^{a+h}_{b+g}]\ \bar\vartheta^4[^{a-h}_{b-g}]\ \bar\vartheta[^{1+h}_{1+g}]\ \bar\vartheta[^{1-h}_{1-g}].$$

At genus-2 h and g become $\vec h = (h,\ \tilde h)$ and $\vec g = (g,\ \tilde g)$ in correspondence with the "electric" and "magnetic" charge shifts.

$$\bar\vartheta[^{a}_{b}\,^{+}_{+}\,^{h}_{g}](\bar\tau) \quad \to \quad \bar\vartheta\begin{bmatrix}\vec a\ +\ \vec h\\ \vec b\ +\ \vec g\end{bmatrix}(\bar\tau^{ij}).$$

Then, the non-perturbative multiplicities are generated by

$$F\begin{bmatrix}\vec h\\ \vec g\end{bmatrix} \ = \ \Phi(\bar\tau^{ij})\ \bar\chi\begin{bmatrix}\vec h\\ \vec g\end{bmatrix}(\bar\tau^{ij}),$$

where $\Phi(\bar\tau^{ij})$ is the $N = 4$ multiplicity function

• The expansion of the genus-2 result is similar to the $\mathcal{N} = 4$ case.

String thresholds and SS supersymmetry breaking

• String threshold corrections in vacua with spontaneously broken susy, have rather different properties.

• The calculations can be done either by integrating one-loop amplitudes

Kaplunovsky-87, Dixon+Kaplunovsky+Louis-91

or using the background field formalism

Kounnas+Kiritsis-94, Petropoulos+Rizos-96
Kiritsis+Kounnas+Petropoulos+Rizos-96, Bachas+Fabre-96

• Unlike the linear behavior in the decompactification volume, a logarithmic behavior is observed

Kiritsis+Kounnas+Petropoulos+Rizos-96-98

$$\lim_{T_2\to\infty} \quad \frac{1}{g^2} \quad \sim \quad \log T_2 + \mathcal{O}\left(e^{-T_2}\right)$$

• There are similar universality $\mathcal{N}=2$ properties as in the case of usual orbifolds.

• The non-perturbative prepotential in the $\mathcal{N}=2$ case can be calculated by duality techniques. No known technique for general non-perturbative SS breaking.

Supersymmetry breaking in orientifolds

• The qualitative features of supersymmetry breaking by gaugino condensation in the heterotic string are similar to SS supersymmetry breaking along the eleventh dimension

Antoniadis+Quiros-97

• This led to SS supersymmetry breaking on orientifolds

Antoniadis+Dudas+Sagnotti-98
Antoniadis+D'Appollonio+Dudas+Sagnotti-98
Angelantonj+Antoniadis+Förger-99

Two options:

♠Translations parallel to branes break susy in the open sector

♠Translation transverse to branes leaves massless brane spectrum supersymmetric.

• Alternative sources of susy breaking: internal magnetic fields on D-branes

Bachas-96
Angelantonj+Antoniadis+Dudas+Sagnotti-00
Blumenhagen+Goerlich+Kors+Lüst-00

Epilogue

- Spontaneous supersymmetry breaking in string theory followed the cue of Scherk and Schwarz in 1979

- Its original formulation was tied to the idea of an internal flux

- It was later realized it is equivalent to freely acting orbifolds.

- In conjunction with BPS techniques, it gave insights into the non-perturbative structure of spontaneously broken extended supergravity theories.

- It is one of the two most popular mechanisms of breaking susy in string theory.

- It is one of the successful examples of the interplay between string theory and supergravity.

Detailed plan of the presentation

- Title page 0 minutes
- Plan 1 minutes
- Ouverture 3 minutes
- The supersymmetry algebra and BPS states 7 minutes
- BPS states and non-perturbative physics 9 minutes
- Helicity supertraces 12 minutes
- The BPS mass formula:the $\mathcal{N}$=4 paradigm 19 minutes
- An N=2 truncation 22 minutes
- Heterotic/type-II duality and branes 24 minutes
- Matching the type-II BPS spectrum 25 minutes
- The magnetic spectrum and the duality 27 minutes
- Spontaneous supersymmetry breaking (à la Scherk-Schwarz) 33 minutes
- Spontaneous versus hard supersymmetry breaking 35 minutes
- Generic partial spontaneous susy breaking 37 minutes