Grisaru –Paris, Oct. 2006

$N = 1$ SUPERSPACE ACTIONS

Conventions

Coordinates:

$$\theta^\alpha \quad , \quad \bar{\theta}^{\dot{\alpha}} \quad , \quad x^{\alpha\dot{\alpha}}$$

Spinor quantities:

$$\psi^\alpha \quad , \quad \lambda_\alpha \quad , \quad \bar{\psi}^{\dot{\alpha}}.......$$

$$\psi.\lambda = \psi^\alpha \lambda_\alpha = \psi^\alpha \lambda^\beta C_{\beta\alpha} \quad , \quad C_{\alpha\beta} = i\epsilon_{\alpha\beta} \quad , \quad \psi^2 = 1/2 \psi^\alpha \psi_\alpha$$

(Same for dotted quantities - always NW-SE contractions)

$$\xi^\alpha \xi^\beta = C^{\alpha\beta} \xi^2$$

Berezin integration

$$\int d\theta_\beta \; \theta^\alpha = \delta_\beta{}^\alpha = \partial_\beta \theta^\alpha |_{\theta=0}$$

integration same as differentiation

Superfields:

$$\Psi(x,\theta,\bar\theta) \quad , \quad \Phi_\alpha(x,\theta,\bar\theta) \quad , ...$$

Transform covariantly under SUSY.

$$\Psi'(x',\theta',\bar\theta') = \Psi(x,\theta,\bar\theta)$$

or

$$\delta\Psi = i[(\epsilon^\alpha Q_\alpha + \bar\epsilon^{\dot\alpha}\bar Q_{\dot\alpha}) \ , \ \Psi]$$

$$Q_\alpha = i(\partial_\alpha - \frac{1}{2}\bar\theta^{\dot\alpha} i\partial_{\alpha\dot\alpha}) \quad , \quad \bar Q_{\dot\alpha} = i(\bar\partial_{\dot\alpha} - \frac{1}{2}\theta^\alpha i\partial_{\alpha\dot\alpha})$$

Therefore $\int d^4x d^4\theta F(\psi,\xi....)$ invariant under SUSY

Covariant derivatives

$$D_\alpha = \partial_\alpha + \frac{1}{2}\bar\theta^{\dot\alpha} i\partial_{\alpha\dot\alpha} \quad , \quad \bar D_{\dot\alpha} = \bar\partial_{\dot\alpha} + \frac{1}{2} i\partial_{\alpha\dot\alpha}$$

$$\{D_\alpha, \bar D_{\dot\beta}\} = i\partial_{\alpha\dot\beta}$$

Chiral/antichiral superfields

$$\bar D_{\dot\alpha}\phi = 0 \quad , \quad D_\beta\bar\chi = 0$$

In a θ expansion higher components are derivatives of lower ones

Chiral representation

By similarity transformation on all quantities,

$$D_\alpha = \partial_\alpha + \bar{\theta}^{\dot{\alpha}} i \partial_{\alpha\dot{\alpha}} \quad , \quad \bar{D}_{\dot{\alpha}} = \bar{\partial}_{\dot{\alpha}} \quad , ...$$

Then chiral superfield

$$\Phi(x, \theta) = A + \theta^\alpha \psi_a + \theta^2 F$$

only. (Then antichiral field more complicated, higher components contain derivatives of lower components)

$$\bar{\phi} = e^{i\theta^\alpha \bar{\theta}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}}} [\bar{A} + \bar{\theta}^{\dot{\beta}} \bar{\psi}_{\dot{\beta}} + \bar{F} \bar{\theta}^2]$$

Will generalize this.

Components by projection

If $\Psi = C + \theta^\alpha \chi_\alpha + ... + \theta^\alpha \bar{\theta}^{\dot{\alpha}} A_{\alpha\dot{\alpha}} + ... + \theta^2 \bar{\theta}^2 \tilde{D}^2$

then

$$C = \Psi|_{\theta=\bar{\theta}=0} \quad , \quad \chi_\alpha = D_\alpha \Psi|_{\theta=\bar{\theta}=0} \quad , ... \; \tilde{D} = D^2 \bar{D}^2 \Psi|_{\theta=\bar{\theta}=0}$$

WZ action

$$\int d^4xd^4\theta\bar{\phi}\phi + \int d^2\theta[v\phi + 1/2m\phi^2 + \lambda/3!\phi^3] + h.c.$$

More general

$$\int d^4xd^4\theta K(\bar{\phi}\phi) + \int d^4xd^2\theta W(\phi) + h.c.$$

How to go to components?

a)By direct expansion in thetas and Berezin integration.
b) Replace integration by differentiatiion
e.g.

$$\int d^4xd^4\theta\ \bar{\phi}\phi = \int d^4x\ \partial^{\dot{\beta}}\partial_{\dot{\beta}}\ \partial^\alpha\partial_\alpha\ (\bar{\phi}\phi)$$

Under $\int d^4x$ replace ∂_α by $D_\alpha = \partial_\alpha + \frac{1}{2}\bar{\theta}^{\dot{\alpha}} i\partial_{\alpha\dot{\alpha}}$

$$\rightarrow \int d^4x\ \bar{D}^2D^2(\bar{\phi}\phi) = \int d^4x\ \bar{D}^2(\bar{\phi}D^2\phi)$$
$$= \int d^4x\ [(\bar{D}^2\bar{\phi})(D^2\phi) + (\bar{D}^{\dot{\alpha}}\bar{\phi})(\bar{D}_{\dot{\alpha}}D^2\phi) + \bar{\phi}(\bar{D}^2D^2\phi)]$$

with everything evaluated at $\theta = 0$.

Define components by projection:

$$\phi| = A\ \ ,\quad D_\alpha\phi| = \psi_\alpha\ \ ,\quad D^2\phi| = F$$

Use commutation relations for D's, get

$$\int d^4x[\bar{F}F + i\bar{\psi}^{\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\psi^\alpha + \bar{A}\Box A]$$

Gauge fields

$$\bar{\phi}\phi \quad invariant \;\; under \quad \phi \to e^{i\lambda}\phi \quad , \quad \bar{\phi} \to \quad \bar{\phi}e^{-i\lambda}$$

for λ constant matrix. Generalize to Λ chiral.

$$\bar{\phi}\phi \to \bar{\phi}e^{-i\bar{\Lambda}}e^{i\Lambda}\phi$$

not invariant.

Introduce gauge prepotential V such that

$$e^V \to e^{i\bar{\Lambda}}e^V e^{-i\Lambda}$$

Then $\bar{\phi}e^V\phi$ invariant .

$$W_\alpha = \bar{D}^2 e^{-V} D_\alpha e^V$$

gauge invariant field strength - **chiral**. SSYM action

$$\int d^4x d^2\theta W^\alpha W_\alpha \quad \left(+ \int d^2\bar{\theta}\bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}} \quad \right)$$

second term differs from first by total derivative.

Later will expand in components by projection

Covariant approach

Introduce gauge covariant derivatives ∇_α , $\overline{\nabla}_{\dot\alpha}$ such that $\nabla_\alpha\phi$ transforms like ϕ.

Generally

$$\nabla_A = (\nabla_\alpha \ , \ \overline{\nabla}_{\dot\alpha} \ , \ \nabla_{\alpha\dot\alpha}) = D_A - i\Gamma_A$$

$$[\nabla_A, \nabla_B\} = T_{AB}{}^C\nabla_C$$

but form of $T_{AB}{}^C$ restricted by *constraints*, e.g.

$$\{\nabla_\alpha, \overline{\nabla}_{\dot\alpha}\} = i\nabla_{\alpha\dot\alpha} \quad \Longrightarrow \quad T_{\alpha\ \dot\alpha}{}^{\beta} = 0 \quad , \text{etc}$$
$$[\overline{\nabla}_{\dot\alpha}, i\nabla_{\beta\dot\beta}] = -iC_{\dot\beta\dot\alpha}W_\beta$$

and, consequently, *Bianchi identities*, e.g.

$$\nabla^\alpha W_\alpha + \overline{\nabla}^{\dot\alpha}\bar{W}_{\dot\alpha} = o$$

In *gauge chiral representation* $(A = (\alpha, \dot\alpha, (\alpha\dot\alpha))$

$$\nabla_A = \left(e^{-V}D_\alpha e^V \ , \ \bar{D}_{\dot\alpha} \ , \ -i\{\nabla_\alpha, \overline{\nabla}_{\dot\alpha}\}\right)$$

(Can do it symmetrically instead - doesn't matter).

Covariantly chiral/antichiral superfields

Defined such that

$$\overline{\nabla}_{\dot{\alpha}}\Phi = 0 \quad , \quad \nabla_\alpha \bar{\Phi} = 0 \qquad \Longrightarrow \Phi = \phi \quad , \quad \bar{\Phi} = \bar{\phi} e^{-V}$$

(just field redefinitions - gives nice covariant components)

Component actions

SSYM: (since integrand singlet can replace $D \to \nabla$)

$$\int d^4x d^2\theta W^2 = \int d^4x D^2(W^2) = \int d^4x \nabla^2(W^2) = \int d^4x \nabla^\alpha(W^\beta \nabla_\alpha W_\beta)$$
$$= \int d^4x W^\alpha(\nabla^2 W_\alpha) + (\nabla^\alpha W^\beta)(\nabla_\alpha W_\beta)$$

all evaluated at $\theta = 0$

Manipulations:

$$\nabla^2 W_\alpha = -\frac{1}{2}\nabla_/ a \nabla^\beta W_\beta = \frac{1}{2}\nabla_/ a \overline{\nabla}^{\dot{\beta}} \bar{W}_{\dot{\beta}} == -\frac{1}{2} i \partial_{\alpha\dot{\beta}} \bar{W}^{\dot{\beta}}$$

having used Bianchi identity and chirality of W. Also

$$\nabla_\alpha W_\beta = symm. + antisymm = C_{\alpha\beta}\nabla^\delta W_\delta + f_{\alpha\beta}$$

the D-auxiliary field plus the self dual part of gauge field strength. etc. etc.

Proceeding in this fashion find

$$S = \int d^4x d^4\theta \bar{\Phi}\Phi + \alpha\gamma V + \mathrm{tr}\int d^4x d^2\theta W^2 + \int d^4x d^2\theta P(\Phi) + h.c. =$$

$$\int d^4x [A\Box\bar{A} + \psi^\alpha i(\nabla_\alpha)^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}} + i\bar{A}\lambda^\alpha\psi_\alpha - i\bar{\psi}^{\dot{\alpha}}\bar{\lambda}_{\dot{\alpha}}A - \bar{A}\tilde{D}A + F\bar{F}$$

$$+\lambda^\alpha i\nabla_{\alpha\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}} - \frac{1}{2}f^{\alpha\beta}f_{\alpha\beta} + D^2$$

$$+\gamma\tilde{D} + [P'F + \frac{!}{2}P"\psi^\alpha\psi_\alpha + h.c.]$$

(Derivatives automatically covariantized wrt Yang-Mills)

SUPERGRAVITY

Generalized coord. transf. of chiral superfields

$$\Phi \to e^{i\Lambda}\Phi \quad , \quad \Lambda = i\Lambda^M D_M$$

Covariant derivatives and constraints

Vielbein $\quad E_A{}^M(x,\theta) \quad$, Lorentz connections $\quad \Phi_A(x,\theta)$.

Covariant derivatives

$$\nabla_A = E_A + \Phi_A M \ = \ E_A{}^M D_M + \Phi_{A\delta}{}^\gamma M_\gamma{}^\delta + \Phi_{A\dot\gamma}{}^{\dot\delta} M_{\dot\gamma}{}^{\dot\delta}$$

$$[\nabla_A, \nabla_B\} = T_{AB}{}^C + R_{AB\gamma}{}^\delta M_\delta{}^\gamma + h.c.$$

Wess-Zumino constraints solved by Siegel in terms of **prepotentials**

$$H_{\alpha\dot\alpha} \quad , \quad \phi$$

Real vector superfield and scalar chiral compensator (or some equivalent depending whether one wants *minimal* or *nonminimal* N=1 sugra). (Also H_α , $H_{\dot\alpha}$) but can be gauged away.

e.g. *in chiral representation*

$$E_\alpha = e^{-H}\bar\Psi D_\alpha e^H \quad , \quad E_{\dot\alpha} = \Psi \bar D_{\dot\alpha} \quad \text{etc.}$$

Ψ expressible in terms of prepotentials, $H = H^M i D_M$.

Superdeterminant

$$E = \mathrm{sdet} E_A{}^M$$

Sugra action

$$S_{SG} = \int d^4x d^4\theta E^{-1}$$

invariant under supercoord. transf. : sugra action.

More explicitly

$$S_{SG} = -\frac{3}{2\kappa^2} \int d^4x d^4\theta \left(\hat{E}\right)^{-1/3} \left(1.e^{-\overleftarrow{H}}\right)^{\frac{1}{3}} \phi e^{-H} \bar{\phi}$$

$$\hat{E} = \mathrm{sdet} \hat{E}_A^M \quad , \quad \overleftarrow{H} = H^M \overleftarrow{D}_M$$

$$\hat{E}_\alpha = e^{-H} D_\alpha e^H \quad , \quad \hat{E}_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}} \quad , \quad \hat{E}_{\alpha\dot{\alpha}} = \{\hat{E}_\alpha, \hat{E}_{\dot{\alpha}}\}$$

Matter actions

For general matter, e.g chiral superfield kinetic term

$$S = \int d^4xd^4\theta E^{-1}\Phi e^{-H}\bar{\Phi}$$

For chiral terms e.g. superpotential

$$S = \int d^4xd^2\theta\phi^3[\frac{m}{2}\Phi^2 + \frac{\lambda}{3!}\Phi^3]$$

More general, if have Kähler potential $3K(?, e?H??)$ together with sugra and YM

$$S = -\frac{3}{\kappa^2}\int d^4xd^4\theta\ E^{-1}e^{\frac{\kappa^2}{3}K(\Phi,e^{-H}\bar{\Phi})}$$
$$+\int d^4xd^2\theta\ \phi^3[P(\Phi) + W^\alpha W_\alpha]$$

Components

For *covariantly* chiral superfield define again

$$A = \Phi|_{\theta=0} \quad , \quad \psi_\alpha = \nabla_\alpha \Phi|_{\theta=0} \quad , \quad F = \nabla^2 \Phi_{\theta=0}$$

For general actions rewrite first

$$S_{gen} = \int d^4x d^4\theta E^{-1} L_{gen} = \int d^4x d^2\, \theta \phi^3 [\nabla^2 + R] L_{gen}$$

where now integrand is (covariantly) chiral.

For chiral actions get

$$S = \int d^4x d^2\theta \phi^3 L_{ch} = \int d^4x e^{-1} [\nabla^2 + i\bar{\Psi}_{\dot{\alpha}}^{\alpha\dot{\alpha}} \nabla_\alpha + 3\bar{S} + \frac{1}{2} \bar{\Psi}_{\alpha(\dot{\alpha}}^{\dot{\alpha}} \bar{\Psi}^{\alpha}{}_{\dot{\beta}}{}^{\dot{\beta}}]\, L_{ch}|_{\theta=0}$$

(Ψ is gravitino, S is auxiliary field)

Note: For $L_{gen} = 1$ get component sugra action

$$S_{sugra} = \int d^4x e^{-1} [\nabla^2 + i\bar{\Psi}_{\dot{\alpha}}^{\alpha\dot{\alpha}} \nabla_\alpha + 3\bar{S} + \frac{1}{2} \bar{\Psi}_{\alpha(\dot{\alpha}}^{\dot{\alpha}} \bar{\Psi}^{\alpha}{}_{\dot{\beta}}{}^{\dot{\beta}}] R$$

Need to work out $\nabla_\alpha R|$, $\nabla^2 R|$ in terms of gravitino and graviton fields.

There are various ways to obtain these results. e.g. Knutt, M.G., Siegel, hep-th/9711120 ; Gates, Knutt, M.G., Siegel, 9711151

SUSY Guts

For SUSY SU(5) need

$$[10] \sim \begin{bmatrix} \bar{U} & Q \\ & \bar{E} \end{bmatrix} \quad , \quad [\bar{5}] \sim \begin{bmatrix} a & \\ c & \end{bmatrix}$$

$$[H] \sim \begin{bmatrix} H_3 & \\ H_2 & \end{bmatrix} \quad , \quad [\bar{H}] \sim \begin{bmatrix} \bar{H}_3 & \\ \bar{H}_2 & \end{bmatrix}$$

[24]

$$\mathcal{L} = \frac{1}{8g^2} \int d^2\theta \mathrm{Tr} W W$$
$$+ \int d^4\theta \left\{ [10]^\dagger e^{2gV(10)}[10] + \bar{S}^\dagger e^{-2gV^T} \bar{S} + [24]^\dagger e^{2gV(24)}[24] \right.$$
$$\left. + [H]^\dagger e^{2gV}[H] + [\bar{H}] e^{-2gV^T}[\bar{H}] \right\} + \mathcal{L}_{superpotential}$$

vspace1cm

$$\mathcal{L}_{superpotential} = \int d^2\theta \left\{ \lambda^u [H][10][10] + \lambda^d \bar{H}[\bar{5}][10] \right.$$
$$+ m_1 \mathrm{Tr}[24][24] + \lambda' \mathrm{Tr}[24][24][24] + m_2 [\bar{H}][H] + \lambda_2 [\bar{H}][24][H] \}$$