AdS/CFT Correspondence – Selected Topics

SUGRA 30 Paris 2006

I. Correlation Functions in $\mathcal{N}=4$ SYM

II. Holographic RG Flows: Leigh-Strassler flow

AdS/CFT provides precise information about strong coupling limit of 4d gauge theories w/o gravity, info. not otherwise available, from weak coupling (i.e. classical) calculations in 5D gravity.

I. de minimus statement of AdS/CFT duality

IIB stg on $AdS_5 \times S_5$ $\longleftrightarrow$ $\mathcal{N}=4$ SYM, $G = SU(N)$

$N \sim \int_{S_5} F_5$ $\qquad g_s = g_{YM}^2$

$L^4 = 4\pi g_s N \alpha'^2$ $\quad \Leftrightarrow$ equivalent for all values of parameters

Not useful in practice, so one take two limits

1. 't Hooft limit $N \to \infty$ $\quad \lambda = g_s N = g_{YM}^2 N$ fixed

$g_s = \lambda/N \to 0$ weak coupling in stg. thy.

all planar diagrams in gauge thy (since λ is fixed)

2. λ large $\Rightarrow$ $L^2 \gg \alpha'$

low energy or IIB SG limit of stg.

strong coupling in field thy.

A. $D=10$ IIB SG $\quad$ bosons g_{MN}, F_{MNPQR}, ϕ, C, F^a_{MNP}

$G_{10} = 8\pi^4 g_s^2 \alpha'^4$

fermions ψ^α_M, λ^α

Classical $AdS_5 \times S_5$ sol:

$g_s = e^\phi$ constant

$F_5 \sim vol(AdS_5) + vol(S_5)$ $\qquad \int_{S_5} F_5 = N$

$$ds_{10}^2 = \frac{L^2}{(z^0)^2}\eta_{\mu\nu}dz^\mu dz^\mu + L^2 g_{\alpha\beta}d\theta^\alpha d\theta^\beta$$

Poincaré patch on AdS_5 $\qquad$ unit S_5

Isometry gp. $SO(4,2) \otimes SO(6) \sim SU(4)$

Kaluza Klein program: effective 5D SG thy.

with $G_5 = \frac{G_{10}}{Vol(S_5)} = \frac{\pi L^3}{2N^2}$

10D fields have expansions in S_5 harmonics

$$\phi(z,\theta) = \sum \phi_k(z) Y_k(\theta^a)$$

$\mathcal{L}_5$ found to quadratic order by Kim, Romans van N. 1986

(a difficult problem because of mode mixing, dualities)

Fields are grouped in irreps of $SU(2,2/4)$, short irreps because max tensor rank in $h_{\mu\nu} = h_{\nu\mu}$

B. $d=4$ $\mathcal{N}=4$ SYM an $SCFT_4$ with elementary fields A_μ, λ^α, X^i

N×N Hermitean matrices $\quad \alpha = 1, \dots 4 \quad i = 1 \dots 6 \quad SU(4)$

$Tr(\ ) = 0$

Bosonic Sym. Gp! $SO(4,2)_{conf} \otimes SU(4)_R$

$\mathcal{N}=4$ $\quad Q^\alpha, \bar{Q}^{\bar{\alpha}}, S^\alpha, \bar{S}^{\bar{\alpha}}$

$8 + 8 + 8 + 8 = 32$ Supercharges.

Same Super alg. $SU(2,2/4)$

Observables: corr. fns. of gauge inv. composite ops.

Simplest are <u>single</u> <u>trace</u> ops.

$$\mathrm{Tr}\, X^k = \mathrm{Tr}(X^{i_1} X^{i_2} \cdots X^{i_k})_{\text{sym + traceless}}$$

Superconformal primary ops of same short irreps as those of KK fields of SG thy.

$\exists$ 1:1 map between

Single trace ops in $\mathcal{N}=4$ — fields of IIB SG on $AdS_5 \times S_5$

primaries + descendants

$\mathrm{Tr}\, X^k \longleftrightarrow \varphi_k$

$J_i^I \longleftrightarrow A_\mu^I$ — $su(4)_R$ currents

$T_{ij} \longleftrightarrow h_{\mu\nu}$ — stress tensor

* See Below

C. Outline Procedure to calculate CFT corr. fns

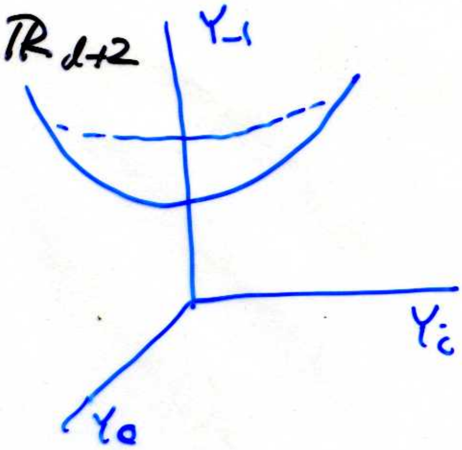
$\langle \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2) \ldots \mathcal{O}_n(x_n)\rangle$ from gravity in AdS_5 bkgd. — Witten 9802150

state program for AdS_{d+1} (to allow for other applics of AdS/CFT).

* All of the above is a consequence of symmetry. We would like to go beyond symmetry to dynamics.

1. Easier to work in $Euc(AdS_{d+1}) = H_{d+1}$

upper half of hyperboloid in $\mathbb{R}_{d+2}$



$$-Y_{-1}^2 + Y_0^2 + \sum_{i=1}^{d} Y_i^2 = -L^2$$

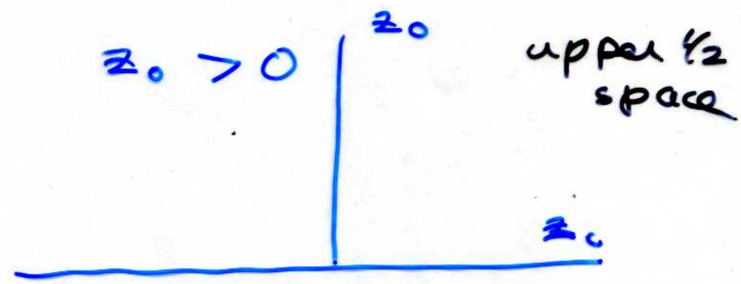
$$z_i = \frac{L Y_i}{Y_0 + Y_{-1}} \qquad z_0 = L^2 / (Y_0 + Y_{-1})$$

Induced metric:

$$ds^2 = \frac{L^2}{z_0^2}\left(dz_0^2 + dz_i^2\right)$$

Conformal to $\mathbb{R}_{d+1}$

$z_0 > 0$



Bdy. plane $z_0 = 0$ is R_d (+ pt at $\infty \to S^d$)

∞ geod. distance from interior, but it can be thought of as a bdy.

2.

2. Free scalar on H_{d+1}:

$$S = \frac{1}{2}\int d^{d+1}z \sqrt{g}\left(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + m^2\varphi^2\right)$$

Wave Eqtn * $\Box\varphi - m^2\varphi = 0$

has (unique) sol which (vanishes as $z_0 \to \infty$)

and approches bdy. as

* $\varphi(z_0,\vec{z}) \xrightarrow[z_0\to 0]{} z_0^{\,d-\Delta}\,\phi(\vec{z})$ for any smooth $\phi(\vec{z})$

$$\Delta = \frac{1}{2}\left(d + \sqrt{d^2+4m^2}\right)$$

This makes sense if $m^2 \geq -d^2/4$ BF bound.

* is a modified Dirichlet bdy. value problem.

Explicit soltn:

$$\varphi(z_0,\vec{z}) = \int d^dx\, K_\Delta(z_0,\vec{z}-\vec{x})\,\phi(\vec{x}) \equiv \varphi_0(z)$$

$$K_\Delta = \frac{\Gamma(\Delta)}{\pi^{d/2}\,\Gamma(\Delta-d/2)}\left(\frac{z_0}{z_0^2+(\vec{z}-\vec{x})^2}\right)^\Delta$$

"Poisson kernel" bulk-to-bdy propagator

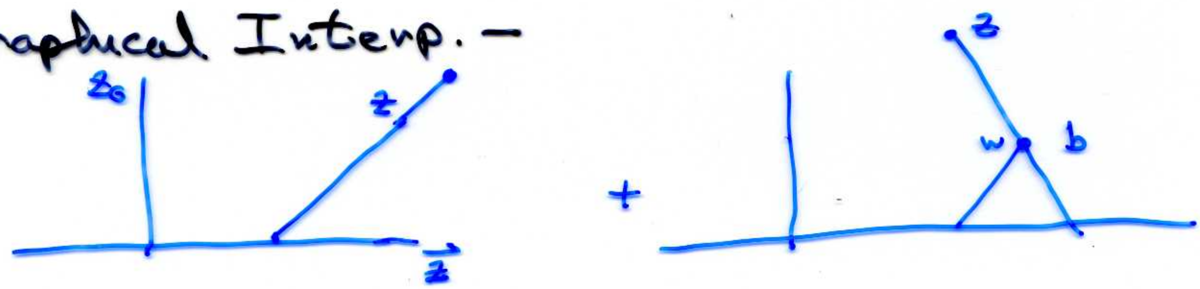
3. Interacting scalar:

$$S = \int\left[\frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}m^2\varphi^2 + \frac{1}{3}b\,\varphi^3 + \dots\right]$$

Iterative soltn: $\varphi(z) = \varphi_0(z) + b\int d^{d+1}w\sqrt{g}\,G(z,w)\,\varphi_0^2(w) + \cdots$

bulk-to-bulk prop. $(\Box - m^2)G = \delta^{d+1}(w,z)$

Graphical Interp. –



4. General Prescription

$\varphi(z)$ classical sol. of $\frac{\delta S}{\delta \varphi} = 0$ with

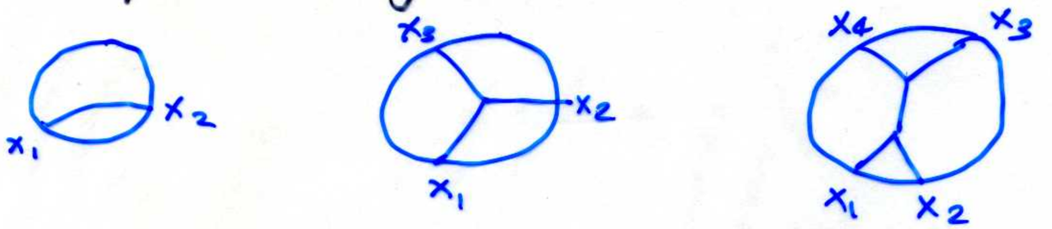
$$\varphi(z) \xrightarrow[z_0 \to 0]{} z_0^{d-\Delta}\, \varphi(\vec{z})$$

On shell action $S[\varphi]$ is fnl. of bdy data

$S[\varphi]$ is generating fnl of CFT correlators!

$$\langle \mathcal{O}(\vec{x}_1) \ldots\ldots \mathcal{O}(\vec{x}_n) \rangle = \frac{\delta}{\delta\varphi(\vec{x}_1)} \cdots \frac{\delta}{\delta\varphi(\vec{x}_n)}\, S[\varphi]$$

5. Graphical algorithm — Witten diagrams.



$+ \cdots$

Rules for calculation + interp.:

a. each bdy. pt. $\vec{x}_i$ is pt. of $\mathbb{R}^d$ where $\mathcal{O}(\vec{x}_i)$ is inserted.

b. each bulk pt. is integrated $\int d^{d+1}w \sqrt{g}$

c. each bulk- bdy line, factor $K_\Delta(z_0, \vec{z} - \vec{x})$

each bulk-bulk line, factor ~~$G(z,w)$~~

$$G(z,w) \sim \left(\frac{1}{u}\right)^\Delta F(a, b, c, -\tfrac{2}{u})$$

u is chordal distance on hyperboloid

d. each n-pt vertex factor b_n from

$$\mathcal{L}_{int} = \frac{1}{n} b_n \varphi^n$$

6. Subtleties:

i. 2pt fns require cutoff + limiting procedure

Gubser, Klebanov Polyakov 9802150

DZF, Mathur, Matusis, Rastelli 9804058

ii $S[\phi]$ is divergent + requires regularization (deep + difficult) Holographic Renormalization

Henningson + Skenderis 9806087

Skenderis et al . . . . .

Practically important for anomalies and 1- and 2-pt. fns. in RG flows.

D. Practical Calc. of CFT correlators:

i. Isometries of AdS_{d+1} in metric: $ds^2 = \frac{1}{z_0^2}\left(dz_0^2 + d\vec{z}^{\,2}\right)$

rotations of z_i	$\frac{1}{2}d(d-1)$	parameters
translations of z_i	d	"
scale: $z_\mu \to \lambda z_\mu$	1	"
special conformal	d	"
$\delta z_\mu = 2c\cdot z\, z_\mu - z^2 c_\mu$		
Total	$\frac{1}{2}d(d+1) = \dim SO(d+1,1)$	

Most useful is discrete sym of

Inversion: $z_\mu = \dfrac{z_\mu'}{(z')^2}$ coord trf. $z_\mu \to z_\mu'$

Inversion $z_\mu = z'_\mu / (z')^2$

On scalar field: $\varphi(z) \longrightarrow \varphi'(z) = \varphi(z')$

On gauge field $A_\mu(z) \longrightarrow A'_\mu(z) = \frac{\partial z'^\nu}{\partial z^\mu} A_\nu(z')$

$$= (z')^2 J_\mu{}^\nu(z') A_\nu(z')$$

$$J_{\mu\nu}(z) = J_{\mu\nu}(z') = \delta_{\mu\nu} - \frac{2 z_\mu z_\nu}{z^2}$$

"Any" gen coo. action on AdS_{d+1}

$$S = \int d^{d+1}z \sqrt{g} \left(\partial^\mu\varphi \partial_\mu \varphi + F_{\mu\nu}F^{\mu\nu} + \cdots \right)$$

is invariant under inversion isometry.

Exercises: $J_{\mu\rho}(z) J_{\rho\nu}(z) = \delta_{\mu\nu}$

$$J_{\mu\nu}(z-w) = J_{\mu\mu'}(z') J_{\mu'\nu'}(z'-w') J_{\nu'\nu}(w')$$

ii) Inversion is conformal isom. of a CFT_d $\vec{x} = \frac{\vec{x}'}{(\vec{x}')^2}$

$$ds^2 = dx_i dx_i = \frac{1}{(x')^4} dx_i' dx_i'$$

$$\frac{d^d x}{|\vec{x}|^d} = \frac{d^d x'}{|\vec{x}'|^d}$$

Scalar op. of dim. Δ trfs. as:

$$\mathcal{O}_\Delta(\vec{x}) \longrightarrow \mathcal{O}'_\Delta(x) = |\vec{x}'|^{2\Delta} \mathcal{O}(x')$$

Conserved current: $V_i(\vec{x})$ has $\Delta = d-1$

$$V_i(\vec{x}) \longrightarrow V_i'(\vec{x}) = |x'|^{2(d-1)} J_{ij}(\vec{x}') V_j(\vec{x}')$$

Correlation fns. trf. with appropriate factors:

$$\langle \mathcal{O}_{\Delta_1}(\vec{x}_1) \ldots \mathcal{O}_{\Delta_2}(\vec{x}_2) \rangle = |\vec{x}_1'|^{2\Delta_1} \cdots |\vec{x}_n'|^{2\Delta_n} \langle \mathcal{O}_{\Delta_1}(\vec{x}_1') \ldots \mathcal{O}_{\Delta_n}(x_n') \rangle$$

iii) Inversion + Other syms. of $\mathbb{R}^d$ restrict form of 2 + 3 pt fns.

$$\langle \mathcal{O}_\Delta(\vec{x})\, \mathcal{O}_{\Delta'}(\vec{y})\rangle = c_\Delta\, \delta_{\Delta\Delta'} \frac{1}{(\vec{x}-\vec{y})^{2\Delta}}$$

$$\langle V_i(\vec{x})\, V_j(\vec{y})\rangle = c_V\, (\partial_i\partial_j - \nabla^2 \delta_{ij}) \frac{1}{(\vec{x}-\vec{y})^{2d-4}}$$

$$\sim c_V \frac{J_{ij}(\vec{x}-\vec{y})}{|\vec{x}-\vec{y}|^{2d-2}}$$

$$\langle \mathcal{O}_{\Delta_1}(x)\, \mathcal{O}_{\Delta_2}(y)\, \mathcal{O}_{\Delta_3}(z)\rangle = \frac{c_{123}}{|x-y|^{\Delta_{12}}\, |y-z|^{\Delta_{23}}\, |z-x|^{\Delta_{31}}}$$

$\Delta_{12} = \Delta_1 + \Delta_2 - \Delta_3$ etc.

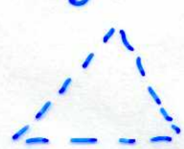
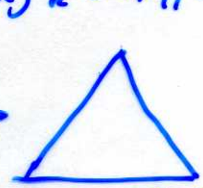
Spacetime dependence completely fixed by sym.

Coeffs: c_Δ, c_V, c_{123} have dynamical content

iv) 3 pt fn of currents $\langle V_i^a(x)\, V_j^b(y)\, V_k^c(z)\rangle$ must be a conformal tensor $f^{abc}\, T_{ijk}(x,y,z)$ which trfs. correctly under inversion + other syms.

In $d=4$, $\exists$ two indep. conf. tensors.

$$T_{ijk}(x,y,z) = c_B\, B_{ijk}(x,y,z) + c_F\, F_{ijk}(x,y,z)$$

$= c_B$  $+ c_F$ 

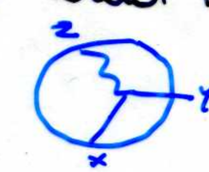
$i V_i = \varphi^* \overleftrightarrow{\partial_i} \varphi$ $\qquad$ $i V_i = \bar{\psi} \gamma_i \psi$

B_{ijk} + F_{ijk} are complicated but specific tensors found from Feynman rules in $\vec{x}$-space

Well defined for $\vec{x} \neq \vec{y} \neq \vec{z}$

v) Witten diags. from AdS/CFT must produce conformal spacetime forms above with specific coeffs which reflect CFT dynamics.

Concentrate on 3 pt fns —

(One can calc. $\langle O_\Delta(x)\, O^*_\Delta(y)\rangle$ from Ward identity for 3 pt fn $\langle J_c(z)\, O_\Delta(x)\, O^*_\Delta(y)\rangle =$ 

)

Witten integral for $\langle O_{\Delta_1}(x)\, O_{\Delta_2}(y)\, O_{\Delta_3}(z)\rangle$

$$A(\vec{x},\vec{y},\vec{z}) = \int \frac{d^{d+1}w}{w_0^{d+1}} \left(\frac{w_0}{(w-\vec{x})^2}\right)^{\Delta_1} \left(\frac{w_0}{(w-\vec{y})^2}\right)^{\Delta_2} \left(\frac{w_0}{(w-\vec{z})^2}\right)^{\Delta_3}$$

? Direct evaluation — combine 3 denoms. + half space restriction $w_0 > 0$ poor method

vi) Instead use inversion of $\vec{x}, \vec{y}, \vec{z}$ w

$$\vec{x} = \vec{x}'/|x'|^2 ,\ \vec{y}, \vec{z} \text{ same.} \qquad w_\mu = \frac{w'_\mu}{(w')^2}$$

$$K_\Delta(w,\vec{z}) = |x'|^{2\Delta} K_\Delta(w', \vec{z}')$$

Result:

$$A(\vec{x},\vec{y},\vec{z}) = |\vec{x}'|^{2\Delta_1} |\vec{y}'|^{2\Delta_2} |\vec{z}'|^{2\Delta_3} A(\vec{x}',\vec{y}',\vec{z}')$$

a Good news — correct scale factors for $\langle O_{\Delta_1} O_{\Delta_2} O_{\Delta_3}\rangle$

$\Longrightarrow$ correct CFT 3-pt fn.

b) bad news same integral in ′ variables.
no progress in doing integral.

vii) Resolution - use transl. sym to move $\vec{z} \to \vec{0}$.

$$A(\vec{x},\vec{y},\vec{z}) = A(\vec{u},\vec{v},0) \qquad \vec{u} = \vec{x}-\vec{z}, \quad v = y-\vec{z}$$

$$\left(\frac{w_0}{(w-\vec{z})^2}\right)^{\Delta_3} \longrightarrow \left(\frac{w_0}{w^2}\right)^{\Delta_3} = (w_0')^{\Delta_3}$$

← no denom. in inverted variables.

Summarize calc.

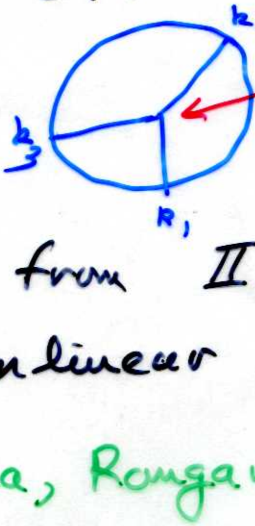
1. Apply inversion to $A(\vec{u},\vec{v},0)$
2. Integral with only two denoms. is quite easy
3. Reexpress in terms of $\vec{x},\vec{y},\vec{z}$ Result

$$A(x,y,z) = \frac{a}{|x-y|^{\Delta_{12}}\,|y-z|^{\Delta_{23}}\,|z-x|^{\Delta_{31}}}$$

$$a = \frac{1}{2}\pi^{d/2}\,\frac{\Gamma(\frac{1}{2}\Delta_{12})\,\Gamma(\frac{1}{2}\Delta_{23})\,\Gamma(\frac{1}{2}\Delta_{31})}{\Gamma(\Delta_1)\,\Gamma(\Delta_2)\,\Gamma(\Delta_3)}\,\Gamma\!\left(\tfrac{1}{2}(\Delta_1+\Delta_2+\Delta_3)-d\right)$$

viii) Consequences for $\mathcal{N}=4$ SYM thy:

$\langle \mathrm{Tr}\,X^k\ \mathrm{Tr}\,X^l\ \mathrm{Tr}\,X^m \rangle =$



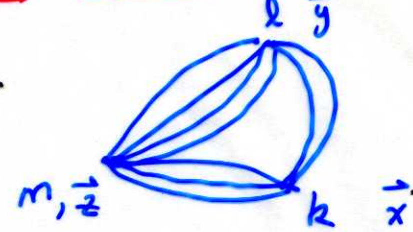
$b_{klm}\,\varphi_k\,\varphi_l\,\varphi_m$

Need cubic couplings b_{klm} from IIB SG on $AdS_5 \times S_5$

Obtained from careful nonlinear KK reduction

Minwalla, Rangamani, Seiberg
9806074

Result: $\langle \mathrm{Tr}\,X^k\ \mathrm{Tr}\,X^l\ \mathrm{Tr}\,X^m \rangle$ from AdS/CFT agrees perfectly with free field calc in $\mathcal{N}=4$ SYM. Product of scalar propagators.



Result should agree with $\lambda = g^2 N$ limit of gauge thy, actually agree with $\lambda = 0$ value

Bold conjecture MRS: 3pt fns are not renormalized all radiative corrections cancel. Amplitude is "protected".

After flurry of activity from many authors, Overwhelming evidence + (perhaps) a proof.

A non-trivial + unexpected property of $\mathcal{N}=4$ SYM obtained from AdS/CFT!

Curiosity: 3 pt fn of $SU(4)$ flavor currents

$$\langle V_i^a(x)\, V_j^b(y)\, V_k^c(z)\rangle \;=$$

Ingredients: 1. bulk to bdy prop for $A_\mu^a(w)$

$$G_{\mu i}(w,\vec{x}) = \frac{3}{\pi}\,\frac{z_0^{\,d-2}}{[z_0^2+(\vec{z}-\vec{x})^2]^{d-1}}\; J_{\mu i}(z-\vec{x})$$

2. bulk YM vertex $\quad f^{abc} A_\mu^a A_\nu^b \partial^\mu A^{c\nu}$

Result — after use of inversion

$$\langle V_i^a V_j^b V_k^c\rangle = \# f^{abc}\left[6\, B_{ijk}(x,y,z) + 4\, F_{ijk}(x,y,z)\right]$$

$\hookrightarrow = 6$  $+\ 4$ 

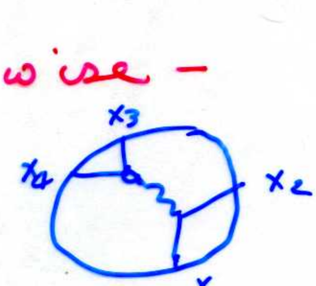
Correct weights for $6\ X^i$ and $4\ \lambda^a$ fields of $\mathcal{N}=4$!

Input to calc. was purely bosonic bulk YM thy.,

but result gives an amplitude which requires boson + fermion content of $\mathcal{N}=4$ SUSY!

AdS would have failed otherwise —

AdS/CFT for 4 pt functions:



One can calculate anom. dim of double trace ops:

$$\mathcal{O}_{kl} = :\mathrm{Tr}X^k\,\mathrm{Tr}X^l:$$

$$\Delta_{kl} = k + l + \frac{\gamma_{kl}}{N^2}$$

strong coupling prediction of AdS/CFT

II. Holographic RG flows:

A. Important to consider deformations of CFT's

$$\mathcal{L} = \mathcal{L}_{CFT} + \sum g_i \mathcal{O}_i$$

Relevant deformations produce RG flows:

$$CFT_{UV} \xrightarrow{\quad g_i(\Lambda) \quad} CFT_{IR}$$

couplings flow:

Correlators:

$$\langle \mathcal{O}(\vec{x})\, \mathcal{O}(\vec{y}) \rangle = f(|\vec{x}-\vec{y}|, g_i) \quad \xrightarrow{x-y\to 0} \frac{C_{UV}}{|\vec{x}-\vec{y}|^{2\Delta_{UV}}}, \qquad \xrightarrow{x-y\to\infty} \frac{C_{IR}}{|x-y|^{2\Delta_{IR}}}$$

B. What is gravity dual of this situation?

5D thy of $g_{\mu\nu}$, φ_i

Girardello, Petrini, Porrati Zaffaroni 9810126

Distler Zamora 9810206

$$S = \frac{1}{8\pi G_5} \int d^5x \sqrt{g} \left[\frac{1}{2} R - \frac{1}{2} (\nabla \varphi_i)^2 - V(\varphi_i) \right]$$

Our previous AdS_5 metric can be rewritten as

$$ds^2 = e^{2r/L} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 \qquad \frac{z_0}{L} = e^{-r/L}$$

This has full $SO(4,2)$ sym, so we now look for a more general 5D soln with Poincaré$_4$ sym to match symmetry of deformed CFT.

$$ds^2 = e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2$$
$$\varphi_i = \varphi_i(r)$$

domain wall ansatz.

In UV limit $r \longrightarrow \infty$ we expect

$$A(r) \longrightarrow r/L_{UV} \qquad \varphi_c(r) \longrightarrow e^{(\Delta_{UV}-d)r/L_{UV}}\, g_c(UV) + \varphi_c(UV)$$

Asymp. AdS at bdy. with scale L_{UV}

Couplings ⟶ values at UV fixed pt.

In IR limit, $r \longrightarrow -\infty$

$$A(r) \longrightarrow r/L_{IR} \qquad \varphi_c(r) \longrightarrow e^{(\Delta_{IR}-d)r}\, g_c(IR) + \varphi_c(IR)$$

deep interior of AdS with different scale L_{IR}

couplings at IR fixed pt values

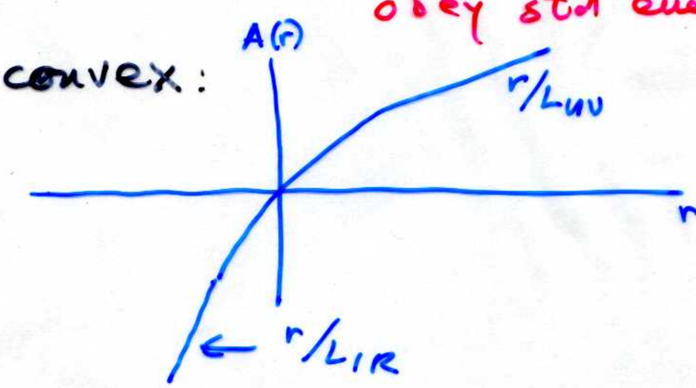
C. From Einstein EOM's one easily derives:

$$R^r_r - R^t_t = 8\pi G_5 \left(T^r_r - T^t_t\right) = 16\pi G_5\, \varphi'^2$$

$$= -3A''$$

So $A'' = \dfrac{-16\pi G_5}{3}\varphi'^2 \le 0$ ← holds for all bulk matter sources which obey std energy cond. of GR.

⟹ Scale factor is convex:



$L_{IR} < L_{UV}$

Field thy meaning from holog. trace anomaly Henningson Skenderis

$$c(r) = a(r) = \frac{\pi}{8G_5}\frac{1}{(A'(r))^3}$$

← $A'' \le 0 \Rightarrow$ $c(r)$ is monotone decreasing toward IR.

Holographic c-thm:

$$C_{UV} = \frac{\pi}{8G_5}(L_{UV})^3$$

$$C_{IR}/C_{UV} = \left(\frac{L_{IR}}{L_{UV}}\right)^3 < 1$$

$$C_{IR} = \frac{\pi}{8G_5}(L_{IR})^2 < C_{UV}$$

hard to prove by field thy methods, easy in AdS/CFT.

D. Leigh-Strassler Flow in $\mathcal{N}=4$ SYM

$\mathcal{N}=1$ Superfields V, Φ^i $i = 1, 2, 3$

$$\mathcal{L} = \underbrace{\int d^4\theta \, \mathrm{Tr}(\Phi^{i\dagger} e^{gV} \Phi^i) + \int d^2\theta \left(\mathrm{Tr}(\Phi^3[\Phi^1, \Phi^2]\right)}_{\mathcal{N}=4} + \frac{1}{2} m \, \mathrm{Tr}(\Phi^3)^2 \Big)$$

relevant perturbation
$\mathcal{N}=4 \rightarrow \mathcal{N}=1$

L-S argue that deformed thy flows to an $\mathcal{N}=1$ $SCFT_4$, superalg. $SU(2,2|1)$ contains $U(1)_R$.

a) By inspection of $\mathcal{L}$ $r_1 = r_2 = 1/2$, $r_3 = 1$ $\qquad r_W = 2$

b) A $\chi SF \longrightarrow$ short rep of $SU(2,2|1)$ with $\Delta = \frac{3}{2} r$

For $\Phi^{1,2,3}$ we have $\Delta_i = 1 + \gamma_i = \frac{3}{2} r_i$

c) Central charge from $U(1)_R$ anomalies — Anselmi, DZF, Grisaru, Johansen 9708042

$$a = \frac{9}{32}(N^2-1)\left(1 + \sum_i (r_i - 1)^3\right)$$

$a_{UV} = \frac{1}{4}(N^2-1)$ since $r_i = \frac{2}{3}$ for all Φ^i

$a_{IR} = \frac{1}{4}(N^2-1)\frac{27}{32}$ from $r_1 = r_2 = \frac{1}{2}$, $r_3 = 1$

E. Supergravity dual: initially from maximal gauged $D=5$ $\mathcal{N}=8$ SG. Gunaydin Romans Warner 1985

<u>Fields</u>: $g_{\mu\nu}$ $A_\mu^{\ a}$ $B_{\mu\nu}$ ϕ_i $\psi_\mu^{\ a}$ χ^{abc}

1 15 12 42 8 48

A thy. with NLσM in scalar sector on coset $\frac{E(6,6)}{USp(8)}$

Complicated $V(\phi_c)$ depending on 40/42 scalars:

1. Find consistent truncation to two scalars $\varphi_2(x)$, $\varphi_3(x)$

Reduced $V(\phi_c)$ related to superpot. $W(\phi_2, \phi_3)$ by

$$V = 18\left(\sum\left(\frac{\partial W}{\partial \varphi_c}\right)^2 - \frac{4}{3}W^2\right)$$

Work with Killing spinor conds: which truncate to

$$\delta\psi_\mu = (D_\mu + \gamma_\mu W(\phi))\,\varepsilon = 0$$

$$\delta\chi_c = \left(\gamma^\mu \partial_\mu \varphi_c - 6\frac{\partial W}{\partial \varphi_c}\right)\varepsilon = 0$$

Stationary pt of V which preserve $\mathcal{N}=1$ SUSY are stationary pts of W: $\frac{\partial W}{\partial \varphi_c} = 0$ $\quad c = 2,3$

a. V has a local max. at $\varphi_2 = \varphi_3 = 0$.

$V_{max} = \frac{-6}{L^2}$ All fields vanish so we have full $SO(6)$ sym. This is the UV fixed pt. which is dual to undeformed $\mathcal{N}=4$ SYM at bdy.

Quadratic terms near max:

$$V \approx \frac{-6}{L^2} + \frac{1}{2}m_2^2\varphi_2^2 + \frac{1}{2}m_3^2\varphi_3^2$$

Exactly what one needs for L-S

$m_2^2 = -4/L^2$ dual to $\mathrm{Tr}\,X^2$ with $\Delta = 2$

$m_3^2 = -3/L^2$ dual to $\mathrm{Tr}\,\psi^2$ with $\Delta = 3$

$$\mathcal{L}_{L.S} = \frac{1}{2} m (\Phi^3)^2 \longrightarrow -\frac{1}{2} m^2 |Z^3|^2 + \frac{1}{2} m \bar{\psi}^3 \psi^3$$

b) V and W have saddle pt at $\phi_2 = \frac{\ln 2}{2\sqrt{3}}$ $\phi_3 = \frac{\ln 3}{2\sqrt{3}}$

$$W_{saddle} = \overline{W} = \frac{2^{2/3}}{3 L_{UV}}$$

These numbers do not look promising, but look at

$$\overline{V} \equiv -18 \cdot \frac{4}{3} \overline{W}^2 = -24 \left(\frac{2^{2/3}}{3 L_{UV}} \right)^2 \equiv -6 \frac{1}{(L_{IR})^2}$$

The AdS sol. at this cp has scale L_{IR} with

$$\left(\frac{L_{IR}}{L_{UV}} \right)^2 = \frac{9}{4 \cdot 2^{4/3}} \dashrightarrow \left(\frac{L_{IR}}{L_{UV}} \right)^3 = \frac{27}{32}$$

SG produces correct trace anomaly of CFT_{IR}!

2. Killing spinor eqtns give BPS flow eqtns

$$* \quad \frac{d\phi_c}{dr} = 6 \frac{\partial W}{\partial \phi_c} \qquad \frac{dA}{dr} = -W(\phi_c(r))$$

$$W = \frac{1}{12 L^2 \rho^2} \left[\cosh^2 \zeta \, (\rho^6 - 2) - 3\rho^6 - 2 \right] \qquad \zeta = \sqrt{2}\, \phi_3 \quad \rho = e^{\frac{1}{2\sqrt{3}} \phi_2}$$

Numerically integrate * to find flow which interpolates between $UV \rightarrow IR$ as discussed above

* free dinner for an analytic sol of *.

3. Calculate quadratic terms for all fields of $D=5$ $\mathcal{N}=8$ SG about IR critical pt.

all 42 ϕ_i, all 48 χ^{abc}, etc.

Find masses of all fluctuations.

All component results can be assembled into $SU(2,2|1)$ multiplets.

Some are short (chiral) mults

$\Delta = \frac{3}{2} r$ e.g. $\mathrm{Tr}(W_\alpha \Phi^i)$ $\Delta = \frac{9}{4}$

Some are conserved current multiplets (fixed Δ) $\Phi^\dagger e^V T^I \Phi$

All results match the field thy.

Others are long multiplets e.g. $\Delta = 1 + \sqrt{7}$

Strong coupling predictions for $CFT_{IR} = CFT_{LS}$

5D work in DZF, Gubser, Pilch, Warner 9904017

Flow in $D=5$ $\mathcal{N}=8$ lifted to $D=10$, IIB SG

Pilch + Warner

A very complete agreement between

$$SG_{5,10} \longleftrightarrow SQFT_4$$