

# THE SUPERSYMMETRIC STANDARD MODEL

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*30 years of supergravity*

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*An introduction*

Is there a  
“SUPERWORLD” ?  
of new particles ?

Could half of the particles (at least)  
have escaped our direct observations ?

# STANDARD MODEL

describes

strong, electromagnetic and weak interactions  
of quarks and leptons

$SU(3) \times SU(2) \times U(1)$  gauge group

spin-1 gauge bosons: gluons,  $\underbrace{W^+, W^-, Z, \text{photon}}$

spin- $\frac{1}{2}$  fermions: quarks and leptons

+ 1 (still unobserved)

spin-0 Englert-Brout-Higgs boson

associated with spontaneous breaking  
of  $SU(2) \times U(1)$  electroweak symmetry

– remarkably successful

– but leaves many questions unanswered

(a long list ...)

Among which

- why fundamental Higgs fields ?  
( do they actually exist ? )

[ many physicists, ignoring supersymmetry, long reluctant to accept fundamental spin-0 fields

→ technicolor → extended technicolor, ... ]

- why a Higgs potential

$$V(\varphi) = \lambda |\varphi|^4 - \mu^2 |\varphi|^2 \quad ?$$

what is the mass of the Higgs boson ?

$$( m_{\text{Higgs}} = \mu \sqrt{2} = v \sqrt{2\lambda} \dots )$$

what fixes  $\mu$  ?

what fixes the coupling constant  $\lambda$  ?

- is the Higgs sector (if exists) the one of the SM or a more complicated one ... ?

- do new particles exist ?

(maybe also new forces ?)

[after LEP, we think we know all (sequential) quarks and leptons]

this has become essential,  
in view of growing evidence for

non-baryonic dark matter

## Other interrogations :

- role of gravity

(related to spacetime through general relativity)

can it be more closely connected with particle physics ?

can one get a consistent theory of quantum gravity ?

question of cosmological constant  $\Lambda$  ...

- can interactions be **unified** ?

approach of grand-unification:

$SU(3) \times SU(2) \times U(1) \subset$  simple gauge group, e.g.  $SU(5)$

$\left\{ \begin{array}{l} \text{gluons} \longleftrightarrow W^\pm, Z, \gamma \quad (+ \text{ other gauge bosons}) \\ \text{quarks} \longleftrightarrow \text{leptons} \end{array} \right.$

with its own questions ...

(Higgs potential and symmetry breaking, origin of hierarchy of mass scales, many coupling constants, relations between  $q$  and  $l$  masses ...)

- can one **relate** particles of different spins ?

etc. ...

We now have a “new” tool,

## SUPERSYMMETRY

BOSONS  $\longleftrightarrow$  FERMIONS

(integer spins)

(half-integer spins)

### SUPERSYMMETRY ALGEBRA:

$$\left\{ \begin{array}{l} \{ Q, \bar{Q} \} = -2 \gamma_{\mu} P^{\mu} \\ [ Q, P^{\mu} ] = 0 \end{array} \right.$$

Gol’fand-Likhtman, Volkov-Akulov, Wess-Zumino, 1970-73

$P^{\mu}$  = generator of space-time translations

*relation with spacetime*

→ *connection with general relativity*

→ *supergravity (1976)*

a new structure

what to do with it ?

## Why was SUSY algebra introduced ?

- Gol'fand and Likhtman, 1970 :

SUSY algebra, written as:

$$\{ Q_L, \bar{Q}_L \} = -2 \gamma_\mu \frac{1+i\gamma_5}{2} P^\mu$$

at the origin of parity non-conservation (in weak interact.) ?

- Volkov and Akulov, 1973 :

is the neutrino a Goldstone particle ?

$$m_\nu = 0 \text{ from SUSY algebra ... ?}$$

( no ... )

V.A. model includes only fermions, no boson ..

( $\delta_{\text{SUSY}}$  fermion = composite bosonic field made of two fermions ...)

**SUSY without bosons !!!**

→

( SUSY without superpartners ... *not the SUSY we know and like ...* )

- Wess and Zumino, 1973 :

Extend to 4 dim. “supergauge” transformations  
acting on  $2d$  string worldsheet

→

Construction of

renormalizable SUSY gauge theories

in 4 dimensions

Can supersymmetry actually be  
a symmetry  
of the fundamental laws of the world ?

*It does not look like it !!!*



(Situation in 1974)

*Nature is “obviously”  
not supersymmetric !*

it seems

1 If supersymmetry:

bosons and fermions would have equal masses:

$$\underline{m_{\text{boson}} \equiv m_{\text{fermion}} !!}$$

Possible answer: break (spontaneously) supersymmetry

But

spontaneous supersymmetry breaking  
did not seem possible !

(in contrast with ordinary symmetries, easy to break)

+ spontaneous breaking of supersymmetry

would generate massless spin- $\frac{1}{2}$  Goldstone fermion

where is the spin- $\frac{1}{2}$  Goldstone fermion of SUSY ?

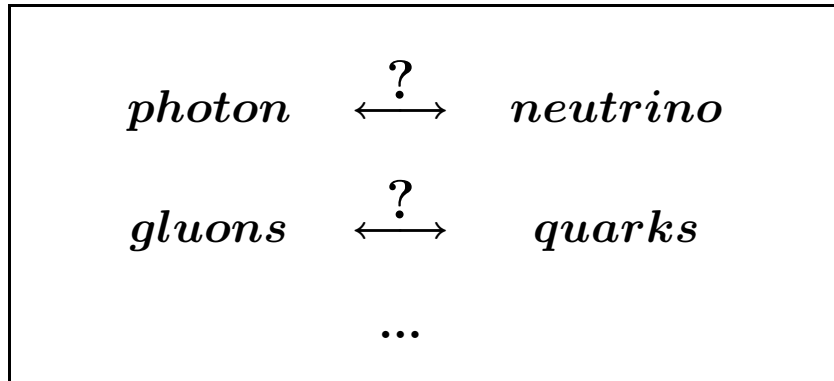
– soft explicit breaking (much easier!) considered very early

(e.g.  $\underline{m_0^2} (\tilde{q}, \tilde{l})$  in 1976 ...)

– “rules of the game”: generate susy-breaking terms spontaneously,  
so that susy may be realized locally (within supergravity, after 1976)

## 2 Which bosons and fermions relate ??

e.g. ??



this does not work ...

## 3 SUSY theories systematically involve

(self-conjugate) Majorana fermions

while Nature only knows Dirac fermions !

## 3' Conserved quantum numbers $B$ and $L$

carried by fermions (quarks and leptons),

not bosons !

this cannot be, in a supersymmetric theory ...

this seemed to make

supersymmetry manifestly irrelevant

to the description of the real world !!

- How to deal with Majorana fermions  
of supersymmetric theories ?
- How to construct the Dirac fermions known in Nature ?
- How to attribute them conserved quantum numbers  
( $B$  and  $L$ ) ?

+ ● How to obtain a correct set of interactions

i.e. weak, electromagnetic and strong interactions

due to the sole exchanges of

$W^\pm$  's,  $Z$  's, photons and gluons,

while avoiding

unwanted exchanges of spin-0 particles ?

This is related with the introduction of

*R*-symmetry and *R*-parity,

in Supersymmetric extensions  
of the Standard Model,

which has essential consequences  
for searches for the new

“supersymmetric particles”

as well as on the nature of the

non-baryonic dark matter

of the Universe

( *neutralinos ...* )

One had to find solutions to all previous problems.

Now that the main answers are,

to a large extent,

known and familiar to everybody,

(and accepted in the community)

they seem rather obvious !

- Let us start with one of the main questions :

Which bosons and fermions may be related ?

\* \* \*

Could one relate the Fermions

constituants of matter

to Bosons, messengers of interactions ?

and arrive to some sort of

Unification between Forces and Matter ??

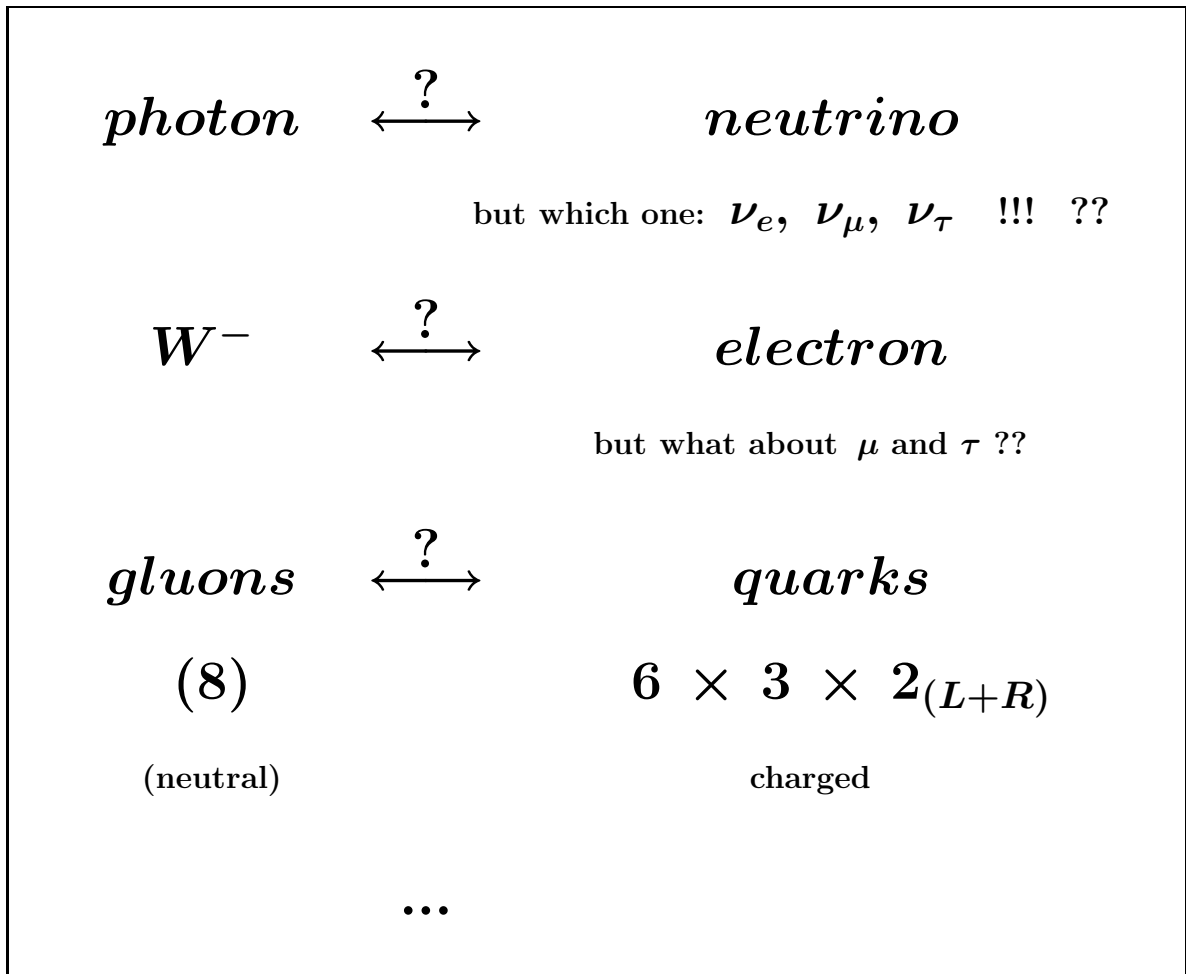
That would be very attractive, but ...

!! ...

Which **BOSONS** and **FERMIONS** may be related?

Are they fundamentally “of the same nature” ?

May be:



this was tried ...

none of these direct associations suitable!

*For example:*

- There is not one neutrino, but three ...

difficult to select one (even a combination)  
to associate to the photon ...

nevertheless, led to developing extended

$N = 2$ , then  $N = 4$ , supersymmetric theories ...

more symmetry ! great ! ( ? )

but: → new (bigger) multiplets; more constraints

hypermultiplets, requiring central charges in algebra

possibility of “mirror particles”, etc.

connection with

extra spacetime dimensions ...

e.g. photon associated with two spin- $\frac{1}{2}$  fermions

(but not neutrinos! — now to be called “photinos”)

and 2 “spin-0 photons”,

now to be viewed as extra degrees of freedom for  
the photon in a 6-dimensional spacetime.

*Not the easiest way to go to a complete, and realistic, theory ...*

*Which degrees of freedom should actually be present in the low-energy theory?*

*We leave this for the time being, and return to ...*



... “simple” ( $\mathbf{N} = \mathbf{1}$ ) supersymmetry ...

What should we do, then ?

- If one neutrino could be associated with photon:

SUSY generator would have to carry  
one unit of the corresponding lepton number.

But it is a self-conjugate Majorana spinor...

How could a real object carry a “charge”, i.e. one unit  
of a conserved additive quantum number ?

*possible solution, and subsequent byproducts:*

Led to introduce a continuous  $R$ -symmetry

acting “chirally” on the supersymmetry generator as:

$$Q \rightarrow e^{-\gamma_5 \alpha} Q$$

(Continuous)  $R$ -symmetry ( $U(1)_R$ ) leads to  
further restrictions on possible (superpotential) interactions

*Not all possible superpotentials are admissible ...*

*Further restrictions to be taken into account,  
in addition to supersymmetry and gauge invariance...*

Continuous  $R$ -symmetry the progenitor of  $R$ -parity ...

→ *a stable dark matter candidate*

- One more reason why the photon cannot be associated with one of the neutrinos:

$$\text{If } \underline{\nu \longleftrightarrow \gamma},$$

$\nu$  interactions would be fixed by gauge invariance + SUSY,

This “ $\nu$ ” would interact with charged particles only,

and proportionally to their electric charge

(not the case for a normal neutrino ( $\nu_e$ ,  $\nu_\mu$  or  $\nu_\tau$ ),

which interacts with  $Z$  ... )

*Such a neutrino could not have weak-neutral-current interactions ...*

This would-be “neutrino”

cannot be identified with  $\nu_e$  (nor  $\nu_\mu$  or  $\nu_\tau$ ),

but should be identified as a new particle,

a “photonic neutrino”

called in 1977 the

“photino”

- What about quarks and gluons?

If quarks associated with gluons,  
SUSY generator would have to carry color, and charge  
transform like a color triplet

Would require a least 3 (or in fact 6 or more ...)

SUSY generators.

( → would require very large multiplets with higher-spin fields, etc. )

*More generally*

- Known fermions ( $q$  and  $l$ ) correspond to

**90** degrees of freedom

(15 chiral fields for each family of  $q$  and  $l$ )

- Known bosons (gluons,  $W^+$ ,  $W^-$ ,  $Z$ ,  $\gamma$ ) :

only **27** degrees of freedom

(+1 if we add the still-undiscovered  
spin-0 Englert-Brout-Higgs boson)

$\implies$  The SUSY idea seems to fail ...

(or new fields are in any case required ...)

We have to accept, and take seriously the idea that

a large number of new fields

should be introduced.

⇒ all particles should be associated with

new superpartners

<i>photon</i>	↔	spin- $\frac{1}{2}$	<i>photino</i>
<i>gluons</i>	↔	spin- $\frac{1}{2}$	<i>gluinos</i>
<i>leptons</i>	↔	spin-0	<i>sleptons</i>
<i>quarks</i>	↔	spin-0	<i>squarks</i>
		...	

this is the famous

“doubling of the number of degrees of freedom”

in susy theories

(within the framework of the “linear realisations” of susy)

SUSY does not relate directly known bosons and fermions !!

but:

Known bosons	↔	New fermions
Known fermions	↔	New bosons

*It now seems obvious, although it was long mocked  
as a sign of the irrelevance of supersymmetry ...*

... + Some additional ingredients

are still needed ...

*encore ... ?!*

*why* ... ?

Early attempt [Nucl. Phys. B 90, 104 (1975)]

at a toy-model relating

{ the photon with a would-be “neutrino”,  
reinterpreted as a photino  
the  $W^-$  with a would-be “electron”,  
reinterpreted as a “wino” or “chargino”

very instructive! It led to a

$SU(2) \times U(1)$  electroweak theory

with electroweak symmetry spont. broken by

a pair of chiral doublet Higgs superfields,  
now known as  $H_1$  and  $H_2$ .

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$$

both left-handed

$$\text{with } \langle h_1^0 \rangle = \frac{v_1}{\sqrt{2}}, \quad \langle h_2^0 \rangle = \frac{v_2}{\sqrt{2}}$$

and a mixing angle (initially called  $\delta$ )

such that

$$\tan \beta = \frac{v_2}{v_1}.$$

## Why two doublet Higgs superfields

rather than a single one ?

- With only one Higgs doublet ( $H_1$ ):

$$\left\{ \begin{array}{l} \text{one charged Dirac "gaugino"} \quad \lambda^- = \lambda_L^- + \lambda_R^- \\ + \text{one chiral charged "higgsino" e.g.} \quad \psi_L^- \end{array} \right.$$

$\implies$  one massive charged Dirac fermion ( $\psi_L^- + \lambda_R^-$ )

one charged chiral fermion ( $\lambda_L^-$ ) would stay massless

- With two Higgs doublets, we get

two charged Dirac fermions,

which may be defined as

$$\left\{ \begin{array}{l} \tilde{W}_1^- = \psi_L^- + \lambda_R^- \\ \tilde{W}_2^- = \lambda_L^- + \psi_R^- \end{array} \right.$$

now known as "charginos"

- We also get a massless Majorana fermion

$$\lambda_\gamma = \sin \theta \lambda_3 + \cos \theta \lambda'$$

partner of the photon field under supersymmetry

$$(A^\mu = \sin \theta W_3^\mu + \cos \theta B'^\mu)$$

now known as "photino"



These charged fermions are

gaugino/higgsino mixings with mass matrix

$$\mathcal{M} = \begin{pmatrix} (m_2 = 0) & \frac{g v_2}{\sqrt{2}} = m_W \sqrt{2} \sin \beta \\ \frac{g v_1}{\sqrt{2}} = m_W \sqrt{2} \cos \beta & \mu = 0 \end{pmatrix}$$

now called “winos” or “charginos”.

This is familiar !

- for the time being:

- no gaugino mass term:  $m_2 = 0$

[ absent in global supersymmetry (unless radiatively generated) ]

- no “ $\mu$  term”

[  $\mu H_1 H_2$  superpotential already replaced by trilinear coupling

$\lambda H_1 H_2 N$  with extra singlet chiral superfield  $N$  ]

as they would violate the continuous  $U(1)_R$  symmetry

- will become allowed once we drop the continuous  $U(1)_R$  symmetry

reducing it to a discrete subgroup of R-parity transformations.

*we shall come back to this later*

Let us now move to the Higgs sector

there must now be

**charged Higgs bosons !**

as well as **several neutral ones**

*“l’horreur” !*

they were almost as badly considered  
as the color-octet of Majorana fermions (“gluinos”)

*(gluinos were believed not to exist,  
as they would violate the general principle of “triality” ...*

*once taken as a constraint to determine  
which particles had a chance to exist,  
and which ones could not possibly exist ... )*

2 Higgs doublets  $\implies$

{ charged Higgses  $H^\pm$ ,  
several neutral ones, now known as  $h, H, A, \dots$

Higgs potential:

$$\begin{aligned}
 V_{\text{Higgs}} &= \frac{\vec{D}^2}{2} + \frac{D'^2}{2} + \dots \\
 &= \frac{g^2}{8} (h_1^\dagger \vec{\tau} h_1 + h_2^\dagger \vec{\tau} h_2)^2 + \frac{g'^2}{8} (h_1^\dagger h_1 - h_2^\dagger h_2)^2 + \dots \\
 &= \frac{g^2 + g'^2}{8} (h_1^\dagger h_1 - h_2^\dagger h_2)^2 + \frac{g^2}{2} |h_1^\dagger h_2|^2 + \dots .
 \end{aligned}$$

$$V_{\text{Higgs}} = \frac{g^2}{8} (\dots)^2 + \frac{g^2 + g'^2}{8} (\dots)^2 + \dots$$

$\rightarrow$  quartic Higgs potential of “MSSM”

quartic Higgs couplings fixed by

$$\frac{g^2 + g'^2}{8} \quad \text{and} \quad \frac{g^2}{2} .$$

We now learn something on “ordinary” particles  
(not superpartners):

**Quartic Higgs coupling(s) no longer arbitrary**

(as in standard model)

fixed by  $SU(2) \times U(1)$  gauge couplings  $g$  and  $g'$

→ many relations (equalities or inequalities) between masses:

between  $m_{H^\pm}$  and  $m_{W^\pm}$ , like  $m_{H^\pm}^2 = m_{W^\pm}^2 \pm \dots$

between masses of neutral Higgses, and  $m_Z$  ... like

$$m_{H \text{ or } h}^2 = m_Z^2 \pm \dots$$

(details of relations depend on how SUSY gets broken)

→ one can also have, in special situations

gauge invariance spontaneously broken with SUSY still conserved,

particles organised in massless or massive multiplets of SUSY

→ massive gauge multiplets of SUSY

$$\left\{ \begin{array}{l} 1 \text{ massive spin-1 gauge boson} \\ 2 \text{ spin-}\frac{1}{2} \text{ inos} \\ 1 \text{ spin-0 Higgs boson} \end{array} \right.$$

→ possibility of relating massive gauge bosons ( $W^\pm$ ,  $Z$ )  
and Higgs bosons ( $H^\pm$  and  $H/h$ )

**gauge boson – Higgs boson unification**

Higgs bosons as  
extra spin-0 degrees of freedom of massive gauge bosons.  
(even without considering extra dimensions, yet ...)

Altogether, all particles should be associated with

new superpartners

<i>photon</i>	$\longleftrightarrow$	<i>photino</i>
<i>gluons</i>	$\longleftrightarrow$	<i>gluinos</i>
<i>leptons</i>	$\longleftrightarrow$	spin-0 <i>sleptons</i>
<i>quarks</i>	$\longleftrightarrow$	spin-0 <i>squarks</i>
	$\dots$	

with, in addition,

Two doublet Higgs superfields  $H_1$  and  $H_2$

for electroweak breaking,

exactly as needed to give

masses to charged leptons and quarks,

through trilinear superpotential couplings  $\propto$ :

{	$H_1 \cdot \bar{E} L$	$\longrightarrow$	charged lepton masses
	$H_1 \cdot \bar{D} Q$	$\longrightarrow$	down quark masses
	$H_2 \cdot \bar{U} Q$	$\longrightarrow$	up quark masses

# Basic ingredients of Supersymmetric Standard Model

(Phys. Lett. 64B (1976) 159; 69B (1977) 489)

- 1)  $SU(3) \times SU(2) \times U(1)$  gauge superfields
- 2) chiral quark and lepton superfields
- 3) two doublet Higgs superfields  $H_1$  and  $H_2$   
for electroweak breaking
- 4) trilinear superpotential for  $q$  and  $l$  masses

- Superpotential constrained to be  
even function of quark and lepton superfields!

includes

$$h_e H_1 \cdot \bar{E} L + h_d H_1 \cdot \bar{D} Q - h_u H_2 \cdot \bar{U} Q$$

- Otherwise: introduces unwanted (and dangerous)

$B$  and/or  $L$  violations!

associated with unwanted exchanges of new spin-0 sparticles!

“ $R$ -parity-violating” superpotential excluded from beginning

*No utility, only a source of problems,  
+ philosophy is to restrict  $\mathcal{L}$  by symmetries,  
not write as many contributions as possible ...*

# Minimal particle content of Supersymmetric Standard Model

Spin 1	Spin 1/2	Spin 0
gluons $g$ photon $\gamma$	gluinos $\tilde{g}$ photino $\tilde{\gamma}$	
$W^\pm$ $Z$	winos $\tilde{W}_{1,2}^\pm$ zinos $\tilde{Z}_{1,2}$  higgsino $\tilde{h}^0$	$H^\pm$ $H$ $h, A$
		$\left. \begin{array}{l} H^\pm \\ H \\ h, A \end{array} \right\} \text{Higgs bosons}$
	leptons $l$ quarks $q$	sleptons $\tilde{l}$ squarks $\tilde{q}$

2 neutral gauginos + 2 neutral higgsinos mix into 4 neutralinos

+ possible additional ingredients

(depending on specific version considered)

additional singlet chiral superfield (“NMSSM”),

with trilinear  $\lambda H_1 H_2 N + \dots$  superpotential

and/or, possibly, extra  $U(1)$  gauge superfield

“hidden sector” associated with supersymmetry-breaking

new “vectorlike” families of quarks and leptons ...

The question of gluino masses

What about our initial  
continuous  $U(1)$   $R$ -symmetry ?

It acts chirally on gluinos

$$g \rightarrow e^{\gamma_5 \alpha} g$$

and would force them to be massless ...

Unbroken continuous  $R$ -invariance

$\implies$

massless **GLUINOS !!!**

which don't seem to exist !!



Massless (or light) gluinos would form light

$R$ -hadrons

$( q \bar{q} \tilde{g} , \quad q q q \tilde{g} )$

expected to decay

$R$ -hadron  $\longrightarrow$  hadrons + neutralino

(1978)

signature:

“missing energy-momentum”  
carried away by unobserved neutralinos

not observed !!

No light  $R$ -hadrons. No light gluinos.

Need mechanism to generate gluino masses.

Requires  $\left\{ \begin{array}{l} \text{SUSY breaking} \\ + \text{absence of continuous } R\text{-invariance} \end{array} \right.$

local SUSY breaking  $\implies$  massive spin- $\frac{3}{2}$  gravitino

gravitino mass term  $m_{3/2}$  (in particular)

breaks continuous  $R$ -invariance, reducing it to  $R$ -parity

(now also allows for gaugino mass terms  $m_{1/2}$ )

Phys. Lett. 70B (1977) 461

Allows for a discrete symmetry to remain unbroken

$$R\text{-parity} = (-1)^R$$

also identified as

$$R_p = (-1)^{2S} (-1)^{3B+L}$$

relates conservation (or non-conservation) of  $R$ -parity  
with conservation (or non-conservation) of  $B$  and/or  $L$  (or  $B - L$ )

if indeed conserved:

- look for **pair-production** (+ decays) of SUSY particles

at colliders

(... , LEP, FNAL, LHC, NLC, ...)

- lightest one (LSP)

probably a neutralino, should be absolutely stable

*lightest neutralino one of the best candidate*

*for the non-baryonic dark matter*

*of the Universe ...*

dark matter searches complementary to accelerator searches

...

- Other essential effects of the new particles:

(superpartners + 2 Higgs doublets + higgsinos)

influence evolution of  $SU(3) \times SU(2) \times U(1)$  gauge couplings

crucial role for

discussion of unification

(cf. next talk)

depending on which particles actually present in “low-energy” spectrum

- SUSY breaking

squark and slepton mass terms  $-\sum_{\tilde{q}, \tilde{l}} m_0^2 (\tilde{q}^\dagger \tilde{q} + \tilde{l}^\dagger \tilde{l})$

considered very early

such terms, + gaugino masses  $m_{1/2}$ , etc.

should be generated spontaneously

(if SUSY algebra is to be realized locally)

How ?

“gravity-induced” SUSY-breaking ?

other “hidden sector” ?

possible contributions from extra  $U(1)$ ’s ?

possible role of extra dimensions ?

- we now consider “soft-breaking terms” as  
parametrisation of our ignorance  
about SUSY breaking mechanism !!
- price to pay: many arbitrary parameters ...

## Extended supersymmetry :

$$N = 1 \quad \longrightarrow \quad N = 2 \quad \longrightarrow \quad N = 4$$
$$\implies \text{R\^ole of possible}$$

extra (compact) dimensions of spacetime

- extremely small ??

$$\sim L_{\text{Planck}} \simeq 10^{-33} \text{ cm} \quad (\text{or fixed by the GUT scale ?}) \quad ???$$

- or significantly larger ?

$$\sim 10^{-16} \text{ or } 10^{-17} \text{ cm} \quad \leftrightarrow \quad \underline{\text{TeV scale?}}$$

The size of extra dimensions

may determine supersymmetry-breaking

and the mass scale of the various superpartners

$$\text{cf.} \quad \underline{m_{3/2}} = \frac{\pi \hbar}{L c} \quad (\text{ or } \frac{1}{2R} ) \quad \text{using}$$

discrete boundary conditions involving R-parity!

$$\rightarrow \text{relations like } m^2(\text{winos}) = m_W^2 + \frac{\pi^2}{L^2}, \quad \text{etc.} \quad (85)$$

*optimistic point of view:*

Both **supersymmetry** and **extra dimensions**  
could show up at particle colliders !!

many scenarios possible

*we would already be happy with supersymmetry!*

We are eagerly waiting for experimental data  
especially from LHC ...