Supergravity gaugings and some string and field theory phenomena

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[Actually, a talk on N=4 supergravity gaugings]

- (Extended) supergravities have many vector fields.
- The ungauged version (all vector fields abelian) can in general be deformed by imposing a non-abelian gauge algebra: "gauged supergravity".
- Then, covariantization of derivatives and interactions, a scalar potential, gravitino mass terms, ...
- A rich phenomenology: spontaneous susy breaking, cosmological constant, masses, ...
- Irrelevant in the visible sector [MSSM]: extended susy does not apply (chirality).
- Relevant in the moduli sector of string theory, in particular N=4 (16 supercharges).

Mostly about N=4, D=4 supergravity, as an effective description of string compactifications with fluxes

- ”Realistic” string constructions are 16 supercharge systems (heterotic, type I, orientifolds of type II, bulk + brane).
- Reduced to D=4, 16 supercharges is N=4 susy.
- Phenomenology requires (the “phenomena”):
  - Supersymmetry breaking, give masses to scalars (moduli problem), ...
  - An acceptable cosmological constant.
  - Appropriate low-energy soft breaking terms.
  - [The standard model] ...

- Supersymmetry broken by compactification, branes, local sources.
- Deformations of "simple" compactifications needed:
  - "Flux compactifications".
  - Difficult to solve the deformed geometry.
  - Why study especially this geometry ?
- Effective, low-energy supergravity approach:
  - N=4, D=4 supergravity deformed by gauging.
  - Parameters of the effective supergravity are the "gauging structure constants".
  - These are also the flux parameters ...

An example of flux superpotential, in a IIA orientifold / Z2 x Z2 orbifold

Kounnas, Petropoulos, Zwirner, JPD.

The orbifold breaks N=4 to N=1.

$$
\begin{aligned}
W \;=\; & \Lambda_{111} + i\,\Lambda'_{111}\;S + i\,\Lambda_{112}\,(T_1+T_2+T_3) - \Lambda'_{112}\;S\;(T_1+T_2+T_3) \\
& + i\,\Lambda_{114}\,(U_1+U_2+U_3) + \Lambda_{113}\,(T_1U_1+T_2U_2+T_3U_3) \\
& - \Lambda_{122}\,(T_1T_2+T_1T_3+T_2T_3) \\
& - \Lambda_{124}\,(T_1U_2+T_1U_3+T_2U_1+T_2U_3+T_3U_1+T_3U_2) - i\;\Lambda_{222}\;T_1T_2T_3
\end{aligned}
$$

For specific flux coefficients, unbroken supersymmetry, anti-de Sitter geometry, the seven moduli stabilized (and massive), six (?) (out of seven) axions stabilized.

[with 14 scalars, the analysis can be performed because supersymmetry does not break.]

Flux coefficients should verify consistency (Jacobi-like) constraints for a consistent gauging !

Possible sources of IIA fluxes [orientifold (D6) / Z2xZ2 orb.] :

- Geometric / twisted tori / Scherk-Schwarz / internal spin connection:

$$-ST_1, -ST_2, -ST_3, T_1U_2, T_2U_2, T_3U_3,$$
$$-T_1U_2, -T_2U_3, -T_3U_1, -T_2U_1, -T_3U_2, -T_1U_3$$

- Form-fluxes: H_3 (NS-NS), F_0, F_2, F_4, F_6 (R-R, with massive IIA parameter and Freund-Rubin flux).

$$iS, iU_1, iU_2, iU_3$$
$$-iT_1T_2T_3, \quad -T_1T_2, -T_2T_3, -T_3T_1, \quad iT_1, iT_2, iT_3, \quad 1$$

The sixteen supercharges algebra dictates conditions on the flux coefficients, either in D=4, N=4 (consistency of the supergravity gauging), or in D=10 (consistency of the string compactification background.

Basics of N=4, D=4 supergravity

- N=4 ungauged: Das / Cremmer, Scherk / Cremmer, Scherk, Ferrara (77-78).
- Gauged: Freedman, Schwarz (78) / Gates, James, Zwiebach (83)
- Vector multiplet couplings, G/H: Chamseddine (81)/ ... / Ferrara, JPD
- Using superconformal calculus [Kaku, Townsend, van Nieuwenhuizen]: de Roo / Bergshoeff, Koh, Sezgin / de Roo, Wagemans, ...

- Supergravity multiplet: 6 vector fields, supergravity dilaton in SU(1,1)/U(1).
- n vector multiplets, each with six real scalars.
- Explicit S-duality from SU(1,1)/U(1).

- A unique scalar sigma-model structure:

$$\frac{SU(1,1)}{U(1)} \times \frac{SO(6,n)}{SO(6)\times SO(n)}$$

- Parameters are "some kind of" structure constants of the algebra gauged by the $6+n$ vector fields: the GAUGING.

- In the formulation given by de Roo, Wagemans, ... , known to be incomplete in the description of its gaugings. dRW: mostly semi-simple (compact or non-compact) gaugings. More can be done, in particular use the dilaton SU(1,1) duality symmetry in the gauge algebra.

- Recent progress using the formalism developed by de Wit, Samtleben and Trigiante: Schön, Weidner (0602024) ...

Electric-magnetic duality

Ungauged supergravities include non-minimal couplings of abelian gauge fields, depending on field strengths only.

Field equations and Bianchi identities

$$\partial^\mu \tilde{G}^A_{\mu\nu} = 0, \qquad \tilde{G}^A_{\mu\nu} \equiv -2\frac{\delta \mathcal{L}}{\delta F^{A\,\mu\nu}}$$

$$\partial^\mu \tilde{F}^A_{\mu\nu} = 0, \qquad \tilde{F}^A_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{A\,\rho\sigma}$$

Electric-magnetic duality: G ⊂ Sp(2m,R) [Gaillard, Zumino]

$$(F^A_{\mu\nu}, G^A_{\mu\nu}) \longrightarrow (F^{A\,\prime}_{\mu\nu}, G^{A\,\prime}_{\mu\nu})$$

Not a symmetry of the Lagrangian,
generates different Lagrangians with equivalent field equations.

Non-trivial scalar - gauge boson couplings lead in general to reduced electric-duality symmetry:

N=8 : $E_{7,7} \subset Sp(56,R)$,

N=4 : $SU(1,1) \times SO(m,6) \subset Sp(12+2m,R)$, [m vector multiplets]

N=4:

SU(1,1) arises in the supergravity multiplet: the true duality symmetry.

SO(m,6) is a global "electric" symmetry of the action.

The gauge algebra is embedded in the duality algebra G.
Gauge bosons can be electric or magnetic.
Scalar couplings of electric and magnetic field strengths are dictated by SU(1,1), and are different.

Gaugings, the embedding tensor

Following de Wit, Samtleben, Trigiante (0212239, 0311225, 0507289,...), Schön, Weidner (0602024), [Petropoulos, Prezas, JPD (06...)].

Gauged generators are linear combinations of electric-magnetic duality generators:

Gauged generator: $$X_M = \Theta_M{}^A \, T_A$$

The embedding tensor

Duality algebra generator

Linear action on gauge fields and magnetic duals: modify the theory to propagate only the original degrees of freedom. Use auxiliary gauge fields (antisymmetric tensors).

Consistency conditions on the embedding tensor:

(were established from explicit studies of various gauged supergravities and in particular of gauging of the maximal theory N=8 [de Wit, Samtleben, Trigiante]).

Firstly, of course, the algebra closes:

$$\boxed{[X_M, X_N] = X_{MN}{}^P X_P = X_{MN}{}^P \Theta_P{}^A T_A}$$

$$[X_M, X_N] = X_{MN}{}^P X_P \quad \longrightarrow \quad (X_M)_N{}^P = -X_{MN}{}^P$$

Closure first implies: $X_{(MN)}{}^P \Theta_P{}^A = 0$

Invariance of symplectic metric: $X_{M[N}{}^Q \Omega_{P]Q} = 0$

Hence, $\hat{X}_{MNP} = X_{MN}{}^Q \,\Omega_{PQ}$ is symmetric in NP.

Closure also leads to quadratic equations (generalizing Jacobi identities). These are hard to solve.

There is a supplementary linear condition, imposed by supersymmetry or more generally from consistency of the gauging procedure (elimination of the supplementary vector boson modes):

$$\boxed{X_{(MN}{}^{Q}\Omega_{P)Q} = 0}$$

The symplectic metric

At the level of the full duality group Sp(2m,R), the embedding tensor is in the product

Fundamental (vector) x Adjoint (symmetric, 2-index)

The condition removes the fully symmetric part.

Easy to solve: defines the embedding tensor (or the X's), up to Jacobi-like identities in terms of certain constant tensors of the duality group, the parameters of the gauged supergravity.

Then, a Lagrangian for this gauged supergravity and a well-defined action of the electric-duality algebra (leading to new equivalent Lagrangians (see Schön, Weidner).

The case of N=4 supergravity

Duality group: G = SU(1,1) x SO(n,6) for n vector multiplets

SU(1,1): $\hat{T}_{\alpha\beta} = \hat{T}_{\beta\alpha}$ SO(n,6): $T_{IJ} = -T_{JI}$

Scalars in coset G/H = SU(1,1)/U(1) x SO(n,6)/SO(n)xSO(6)

Vector fields strengths + duals in rep. (2 , n+6) of G.

Gauging (embedding tensor):

$$X_{\alpha I} = \frac{1}{2}\,\Theta_{\alpha I}{}^{\beta\gamma}\,\hat{T}_{\beta\gamma} + \frac{1}{2}\,\Theta_{\alpha I}{}^{JK}\,T_{JK}$$

Gauged algebra: $[X_{\alpha I}\,,\,X_{\beta J}] = {X_{\alpha I\,\beta J}}^{\gamma J}\,X_{\gamma J}$

Solving the linear constraint leads to

$$\boxed{X_{\alpha I} = \frac{1}{2}{f_{\alpha\,I}}^{JK}T_{JK} + \zeta_\alpha^J T_{IJ} - \zeta_I^\beta T_{\alpha\beta}}$$

Gauging parameters are:

a pair of antisymmetric tensors: $f_{\alpha IJK} = f_{\alpha[IJK]}$

a pair of SO(n,6) vectors: ζ_α^I

Gaugings involving the SU(1,1) e.-m-duality

de Roo et al.: the case $\zeta_\alpha^I = 0$
electric gauging involving SO(n,6) only

Closure constraints (quadratic equations):

$$\zeta_K^\gamma \zeta_\alpha^K = 0$$ [Zero norm SO(n,6) constant vectors]

$$\zeta_{(\alpha}^M f_{\beta)MIJ} = 0$$

$$\epsilon^{\alpha\beta}(2\zeta_{\alpha I}\zeta_{\beta J} + \zeta_{\alpha M} f_{\beta IJM}) = 0$$

$$3 f_{\alpha I[L}{}^K f_{\beta NJ]K} + f_{\alpha LNJ}\zeta_{\beta I} - 3\zeta_{\alpha[L} f_{\beta NJ]I}$$

$$+3\zeta_\alpha^M f_{\beta M[LN}\eta_{J]I} - \zeta_I^\gamma f_{\gamma LNJ}\epsilon_{\alpha\beta} = 0$$

One set of $f_{\alpha\, IJK}$ only: Jacobi identity.

$\zeta_\alpha^I = 0$: Jacobi identities and a mixed condition

$f_{\alpha\, IJK} = 0$: a unique algebra

Electric gaugings:

SU(1,1) on the supergravity dilaton: $S \longrightarrow \dfrac{aS+ib}{icS+d}$

A duality frame can be chosen in which only electric fields are used in gaugings. In this frame, $\hat{T}_{22}$ does not participate in the gauging. Choose then: $\zeta_{1I} = \zeta_I^2 = 0$
Supergravities are in general constructed in this frame.

ζ_{2I} : a zero length vector, unique up to SO(n,6) rotations: one parameter q only.

Axionic shift

Gauge generators:

$$X_{1I} = \tfrac{1}{2} f_{1I}{}^{JK} T_{JK} - \zeta_I^1 T_{11}$$

$$X_{2I} = \tfrac{1}{2} f_{2I}{}^{JK} T_{JK} - \zeta_2^J T_{IJ} - \zeta_I^1 T_{21}$$

SO(1,1)

Higher dimensions: how to get the electric subalgebra of SU(1,1) gauged

Gauge a “symmetry” acting on the dilaton, to incorporate dilaton shift and dilatation symmetries.

$$D_\mu M_{\alpha\beta} = \partial_\mu M_{\alpha\beta} + g A^{M\gamma} \zeta_{(\alpha M} M_{\beta)\gamma} + g A^{M\delta} \zeta_{\epsilon M} \epsilon_{\delta(\alpha} \epsilon^{\epsilon\gamma} M_{\beta)\gamma}$$

$$M_{11} = \frac{1}{\tau_2}, \quad M_{12} = \frac{\tau_1}{\tau_2}, \quad M_{22} = \frac{\tau_1^2 + \tau_2^2}{\tau_2}.$$

$$D_\mu \tau = \partial_\mu \tau + g A^{M1} \zeta_{2M} + g(A^{M2}\zeta_{2M} - A^{M1}\zeta_{1M})\tau - g A^{M2} \zeta_{1M} \tau^2$$

Gauge the SO(1,1) “duality twist”:

- D=5, N=4 pure supergravity, Scherk-Schwarz reduction to D=4 [Villadoro, Zwirner, 0406185]
- Scherk-Schwarz reduction from D=10 (heterotic N=1 case).

An example: Scherk-Schwarz reduction of D=5, N=4 supergravity

D=5: duality group SO(1,1) x SO(5,1).
Six vector fields, in 5 + 1 of global symmetry SO(5).

If obtained by dimensional reduction + truncation from D=10, the 6th vector is the dual of an antisymmetric tensor. Appears then as a "magnetic" vector after reduction D=5 to D=4.

Direct reduction + truncation D=10 to D=4 would produce the electric dual of this vector.

The same gauging uses then the electric or the magnetic gauge field, both sets of $f_{\alpha\, IJK}$ in use.

Scherk-Schwarz reduction using SO(1,1):

The reduction uses a linear combination of the "dilaton SO(1,1)" with a second SO(1,1) ⊂ SO(1,5).

No space for nontrivial $f_{\alpha\, IJK}$

When reduced to D=4: SU(1,1) x SO(6,1), seven vector fields.

Then, a seven-dimensional algebra with a non-compact generator X, five abelian generators in SO(6,1) and the axionic dilaton shift Y :

$$[X_m, Y] = [X_m, X_n] = 0 \qquad [X, Y] = 2qY \qquad [X, X_n] = -qX_n$$

On the Scherk-Schwarz mechanism and N=4 gaugings

Ansatz for generalized dimensional reduction from D=10 to D=4:

$$e_M^A(x,y) = \begin{pmatrix} \delta(x)^\gamma \, e_\mu^a(x) & 2\kappa A_\mu^{\hat{k}}(x)\,\Phi_{\hat{k}}^i(x) \\ 0 & U_{\hat{k}}^{\hat{l}}(y)\,\Phi_{\hat{l}}^i(x) \end{pmatrix}$$

where $\delta(x) = \det \Phi_{\hat{k}}^{\hat{l}}$

Structure constants, verifying Jacobi identities:

$$f_{\hat{k}\hat{l}}{}^{\hat{m}} = (U^{-1})_{\hat{k}}^{\hat{n}}(U^{-1})_{\hat{l}}^{\hat{p}}\left(\partial_{\hat{p}}U_{\hat{n}}^{\hat{m}} - \partial_{\hat{n}}U_{\hat{p}}^{\hat{m}}\right)$$

[U matrix ~ some symmetry of the action]

In other words, introduce non-abelian gauging:

$$\begin{aligned}\omega_M{}^{AB} &= -\frac{1}{2}\left(\partial_M e_N^A - \partial_N e_M^A - f^P{}_{MN} e_P^A\right) e^{NB} \\ &\quad +\frac{1}{2}\left(\partial_M e_N^B - \partial_N e_M^B - f^P{}_{MN} e_P^B\right) e^{NA} \\ &\quad -\frac{1}{2} e^{PA} e^{QB} \left(\partial_P e_Q^C - \partial_Q e_P^C - f^R{}_{PQ} e_R^C\right) e_{MC}\end{aligned}$$

Torsion:

$$S_{MN}^P = \frac{1}{2} f^P{}_{MN}$$

For a D=10 theory reduced to D=4, a six-dimensional gauge algebra induced by (internal) general coordinate transformations.

Under internal gen. coord. transformations

$$\xi^M = (\xi^\mu(x), (U^{-1})_{\hat{l}}^{\hat{k}} \hat{\xi}^{\hat{l}}(x))$$

$A_\mu^{\hat{k}}$ gauge field with structure constants

$\Phi_{\hat{l}}^{\hat{k}}$ scalar field in adjoint rep.

Notice however $\quad \delta_{\hat{\xi}}\, \delta(x) = f_{\hat{k}\hat{l}}{}^{\hat{k}} \hat{\xi}^{\hat{l}}\, \delta(x)$

Either unimodularity condition $\quad f_{\hat{k}\hat{l}}{}^{\hat{k}} = 0$

Or use a non-trivial dilaton shift to compensate the variation:
dilaton symmetries participate in the gauge algebra.

Resulting D=4 gauge algebra has

$$f_{1\,ij\,m'}, \quad f_{2\,IJK} = 0, \qquad \zeta_{2i} \sim f_{ij}{}^j$$

The scalar potential

has a new contribution generated by the trace of the Scherk-Schwarz structure constants

$$V \sim \quad f_{\hat{m}\hat{n}}{}^{\hat{k}} f_{\hat{l}\hat{p}}{}^{\hat{r}} h^{\hat{m}\hat{l}} h^{\hat{n}\hat{p}} h_{\hat{k}\hat{r}} + 2 f_{\hat{m}\hat{n}}{}^{\hat{k}} f_{\hat{k}\hat{p}}{}^{\hat{m}} h^{\hat{n}\hat{p}}$$

$$-4 f_{\hat{m}\hat{k}}{}^{\hat{m}} f_{\hat{n}\hat{p}}{}^{\hat{n}} h^{\hat{k}\hat{p}}$$

[OK with Schön, Weidner]

Counting parameters (heterotic Scherk-Schwarz + H3 flux)

The heterotic string does not generate (perturbatively) both sets of $f_{\alpha\, IJK}$. There are twelve vector fields, 6 vector multiplets.

$f_{2\, IJK} = 0:$ 220 numbers in $f_{1\, IJK}$
Scherk-Schwarz structure constants: 90 numbers
H3 flux: 20 numbers
(110 for other nongeometric fluxes generated acting with dualities of the effective supergravity).

- The dictionary generalized fluxes *vs* generalized structure constants is very useful in studying phenomenology of string vacua.
- Nothing more than an effective field theory approach, but much simpler than working out the geometry.
- The gauge algebras relevant to moduli stabilization, supersymmetry breaking, ... , are subtle. Requires a complete control of supergravity gaugings.
- Can be combined with non-perturbative contributions (which are not however N=4 in general), like condensate superpotentials.