

*"30 years of supergravity"*  
Paris, October 19, 2006

# **superstring realisations of supergravity in ten dimensions**

Carlo Angelantonj  
(Turin University)

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## **SUPERSYMMETRY, SUPERGRAVITY THEORIES AND THE DUAL SPINOR MODEL**

**F. GLIOZZI <sup>\*</sup> and J. SCHERK**

*Laboratoire de Physique Théorique de l'Ecole Normale Supérieure <sup>\*\*</sup>*

**D. OLIVE**

*CERN, Geneva*

*and*

*The Niels Bohr Institute, Copenhagen*

Received 14 January 1977

We study the connection between the dual spinor model and supersymmetry. We show that in the low-energy region, the dual spinor model yields a supersymmetric Yang-Mills theory with  $O(4)$  internal symmetry and a supergravity theory with  $O(4)$  internal symmetry.

The correct model to be considered thus seems to be the  $G = +1$  sector of the NS model together with the Weyl (left-handed) and Majorana (real) sector of the Ramond model. It seems indeed to be trouble-free (no tachyons, no ghosts).

In such a model fermion-fermion and boson-fermion scattering have exactly the same duality properties (see fig. 3).

GSO suggest to arrange the string states according to their two-dimensional statistics

Moreover, (in the light-cone) superstring states carry a representation of the  $SO(8)$  little group

It turns out that the GSO projection arranges states according to their  $SO(8)$  representations

The four  $SO(8)$  conjugacy classes are then in one-to-one correspondence with G-parity

## **Affine $SO(8)_{k=1}$ characters**

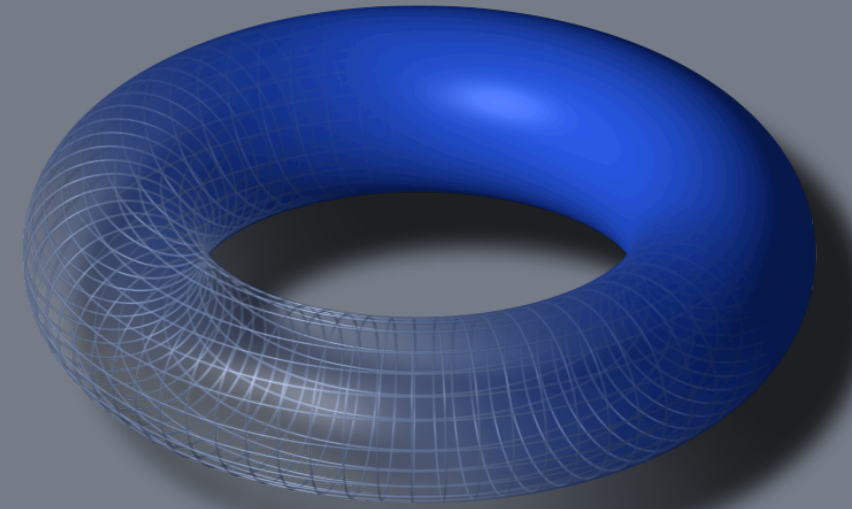
$$\begin{aligned} O_8(q) &= \text{tr}_{\text{NS}} \mathcal{P}_{\text{GSO}}^{(+)} q^{L_0} \\ &= \frac{\theta_3^4(0|\tau) + \theta_4^4(0|\tau)}{2\eta^4(\tau)} \sim \mathbf{1} \end{aligned}$$

$$\begin{aligned} V_8(q) &= \text{tr}_{\text{NS}} \mathcal{P}_{\text{GSO}}^{(-)} q^{L_0} \\ &= \frac{\theta_3^4(0|\tau) - \theta_4^4(0|\tau)}{2\eta^4(\tau)} \sim \mathbf{8}_v \end{aligned}$$

$$\begin{aligned} C_8(q) &= \text{tr}_{\text{R}} \mathcal{P}_{\text{GSO}}^{(+)} q^{L_0} \\ &= \frac{\theta_2^4(0|\tau) + \theta_1^4(0|\tau)}{2\eta^4(\tau)} \sim \mathbf{8}_c \end{aligned}$$

$$\begin{aligned} S_8(q) &= \text{tr}_{\text{R}} \mathcal{P}_{\text{GSO}}^{(-)} q^{L_0} \\ &= \frac{\theta_2^4(0|\tau) - \theta_1^4(0|\tau)}{2\eta^4(\tau)} \sim \mathbf{8}_s \end{aligned}$$

# the torus partition function



$$\mathcal{Z} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \text{tr} \left( \mathcal{P}_{\text{GSO}} q^{L_0} \bar{q}^{\tilde{L}_0} \right)$$

$$= \frac{1}{4} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^{2+4}} \frac{1}{|\eta|^{16}} \sum_{\substack{a,b=0,1 \\ \tilde{a},\tilde{b}=0,1}} C \begin{bmatrix} a & b \\ \tilde{a} & \tilde{b} \end{bmatrix} \frac{\theta^4 \begin{bmatrix} a \\ b \end{bmatrix}}{\eta^4} \frac{\bar{\theta}^4 \begin{bmatrix} \tilde{a} \\ \tilde{b} \end{bmatrix}}{\bar{\eta}^4}$$

space-time momenta

WS bosons

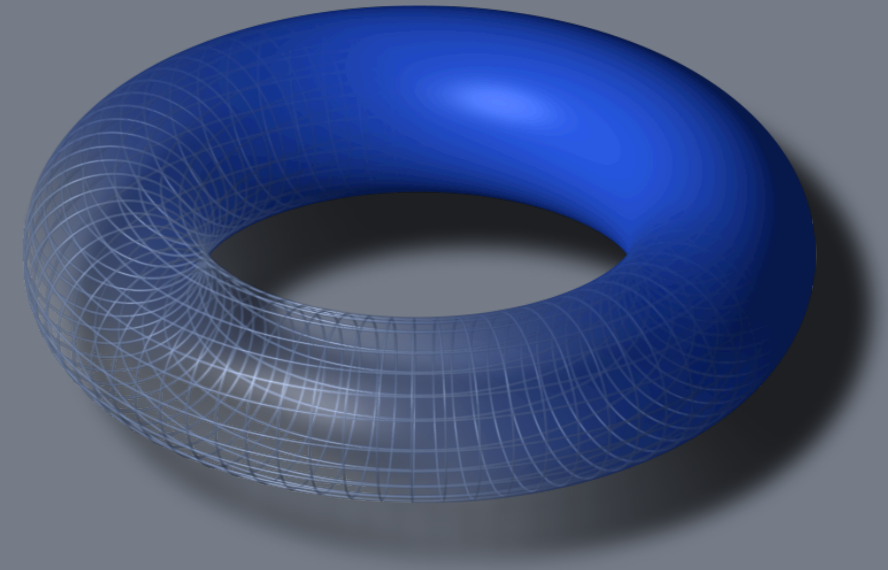
GSO phases

WS fermions

the phases determine the ten-dimensional spectrum,  
thus distinguish among the different superstrings

# ***the torus partition function***

$$\mathcal{Z} = \sum_{i,j=1}^4 \bar{\chi}_i X_{ij} \chi_j$$



$$\chi = (O_8, V_8, -S_8, -C_8)$$

$X_{ij}$  is a matrix of non-negative integers such that

$$S^\dagger X S = X, \quad T^\dagger X T = X$$

GSO phases  $\Leftrightarrow$  modular invariance

# type IIA superstrings

$$\mathcal{T}_{\text{IIA}} \sim (V_8 - S_8)(\bar{V}_8 - \bar{C}_8)$$

$g_{\mu\nu}, \phi, B_{\mu\nu}$      $\psi_R^\mu, \lambda_L$      $\psi_L^\mu, \lambda_R$      $C_\mu, C_{\mu\nu\rho}$

# type IIB superstrings

$$\mathcal{T}_{\text{IIB}} \sim (V_8 - S_8)(\bar{V}_8 - \bar{S}_8)$$

$g_{\mu\nu}, \phi, B_{\mu\nu}$      $\psi_L^\mu, \lambda_R$      $\psi_L^\mu, \lambda_R$      $C_0, C_{\mu\nu}, C_{\mu\nu\rho\sigma}^{(+)}$

# ***maximal supergravities in D=10***

Nahm 1977

***type IIA supergravity:***  $\mathcal{N} = (1, 1)$

Cremmer, Julia, Scherk 1978

Giani, Pernici 1984

Campbell, West 1984

$$\{g_{\mu\nu}, \phi, B_{\mu\nu}, C_{\mu}, C_{\mu\nu\rho}; \psi_{\text{L}}^{\mu}, \psi_{\text{R}}^{\mu}, \lambda_{\text{L}}, \lambda_{\text{R}}\}$$

***type IIB supergravity:***  $\mathcal{N} = (2, 0)$

Schwarz 1983

Howe, West 1984

$$\{g_{\mu\nu}, \phi, B_{\mu\nu}, C_0, C_{\mu\nu}, C_{\mu\nu\rho\sigma}^{(+)}; 2\psi_{\text{L}}^{\mu}, 2\lambda_{\text{R}}\}$$

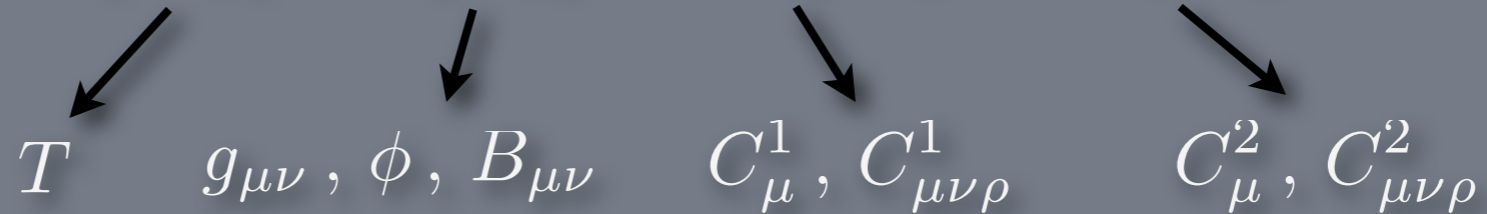
**Although chiral type IIB is free of anomalies**

Alvarez-Gaumé, Witten 1984



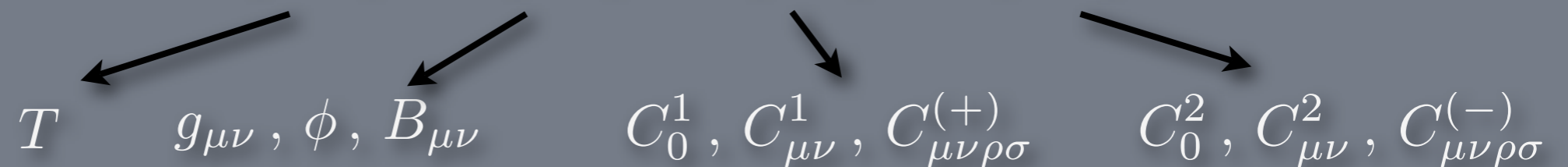
# type 0A superstrings

$$\mathcal{T}_{0A} = |O_8|^2 + |V_8|^2 + \bar{S}_8 C_8 + \bar{C}_8 S_8$$



# type 0B superstrings

$$\mathcal{T}_{0B} = |O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2$$



purely bosonic and tachyonic!

**Heterotic String**

David J. Gross, Jeffrey A. Harvey, Emil Martinec, and Ryan Rohm  
*Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544*  
 (Received 21 November 1984)

A new type of superstring theory is constructed as a chiral combination of the closed  $D = 26$  bosonic and  $D = 10$  fermionic strings. The theory is supersymmetric, Lorentz invariant, and free of tachyons. Consistency requires the gauge group to be  $\text{Spin}(32)/\mathbb{Z}_2$  or  $E_8 \times E_8$ .

$$\mathcal{T}_{\text{SO}(32)} \sim (V_8 - S_8)(O_{32} + S_{32})$$

$$\mathcal{T}_{E_8 \times E_8} \sim (V_8 - S_8)(\bar{O}_{16} + \bar{S}_{16})(\bar{O}_{16} + \bar{S}_{16})$$

to build string states we combine holomorphic states in the NS and R sector together with bosonic oscillators from the anti-holomorphic sector pointing along the space-time directions and along the internal directions

$$(V_8 - S_8) \times \alpha_{-1}^{\mu} \quad \longrightarrow \quad g_{\mu\nu}, \phi, B_{\mu\nu}, \psi_L^{\mu}, \lambda_R$$

$$(V_8 - S_8) \times \alpha_{-1}^I \quad \longrightarrow \quad A_{\mu}^I, \lambda_L^I \quad \begin{array}{l} \text{Spin}(32)/\mathbb{Z}_2 \\ E_8 \times E_8 \end{array}$$

# *minimal supergravity in D=10*

Nahm 1977

$$\{g_{\mu\nu}, \phi, B_{\mu\nu}; \psi_L^\mu, \lambda_R\}$$

Bergshoeff, de Roo, de Wit, van Nieuwenhuizen 1982

## *matter multiplet: super-Yang-Mills*

Gliozzi, Scherk, Olive 1976  
Brink, Scherk, Schwarz 1976  
Chapline, Manton 1983

$$\{A_\mu; \lambda_L\}$$

The only anomaly-free theories have gauge group

Green, Schwarz 1984

$$E_8 \times E_8$$

$$SO(32)$$

$$E_8 \times U(1)^{248}$$

$$U(1)^{496}$$

# ***non-supersymmetric heterotic strings***

$$\mathcal{I} = V_8 (\bar{O}_{16}\bar{O}_{16} + \bar{S}_{16}\bar{S}_{16}) - S_8 (\bar{O}_{16}\bar{S}_{16} + \bar{S}_{16}\bar{O}_{16}) \\ + O_8 (\bar{V}_{16}\bar{C}_{16} + \bar{C}_{16}\bar{V}_{16}) - C_8 (\bar{V}_{16}\bar{V}_{16} + \bar{C}_{16}\bar{C}_{16})$$

Alvarez-Gaumé, Ginsparg, Moore, Vafa 1986

*massless excitations comprise*  $g_{\mu\nu}, \phi, B_{\mu\nu}$

*gauge group*  $O(16) \times O(16)$

*left-handed fermions in the reps*  $(128, 1) + (1, 128)$

*right-handed fermions in the reps*  $(16, 16)$

# ***free-fermionic constructions***

Antoniadis, Bachas, Kounnas, Windey | 1986

Antoniadis, Bachas, Kounnas | 1987

Kawai, Llewellen, Tye | 1987

Lerche, Lüst, Schellekens | 1987

Narain, Sarmadi, Vafa | 1987

At special points of moduli space of toroidal (orbifold) compactifications zero-modes of world-sheet bosons can be fermionised

$$\mathcal{Z} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z_B^2 \sum_{\text{spin str}} C \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} Z_{\text{long},3/2} \begin{bmatrix} a_\psi \\ b_\psi \end{bmatrix} \prod_{f=1}^{64} Z_F \begin{bmatrix} a_f \\ b_f \end{bmatrix}$$

*the phases are determined by demanding one loop modular invariance and two-loop factorisation*

*to summarise ...*

***modular invariance for models of closed oriented strings is the rationale behind the Gliozzi-Scherk-Olive projection***

*it allows to extend string constructions to lower dimensions and to non-geometrical spaces (internal exact/abstract conformal field theories)*

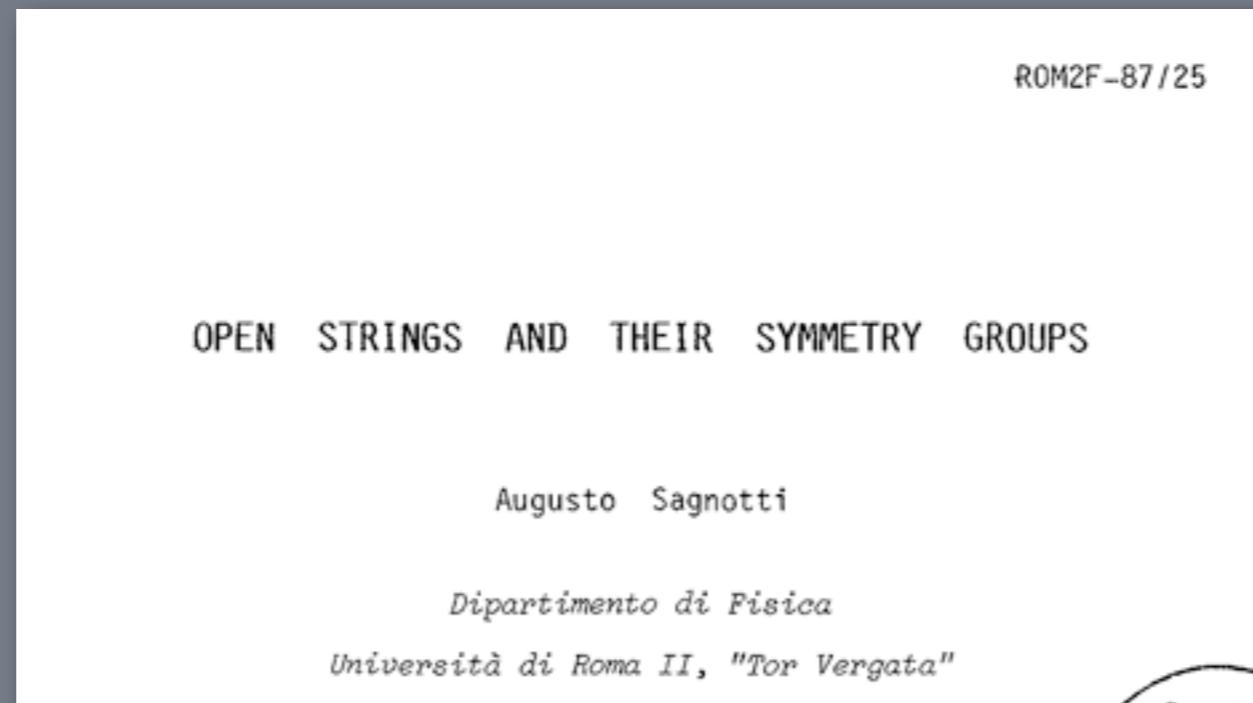
*in turn, modular invariance automatically yields an anomaly-free theory*

# ***What about open strings?***

*open strings alone do not yield a consistent theory  
(they can join ends and form closed strings!)*

*oriented closed strings together with oriented open strings  
do not yield consistent vacuum configurations*

# Orientifold constructions



*a consistent way of tying together  
closed and open superstrings*

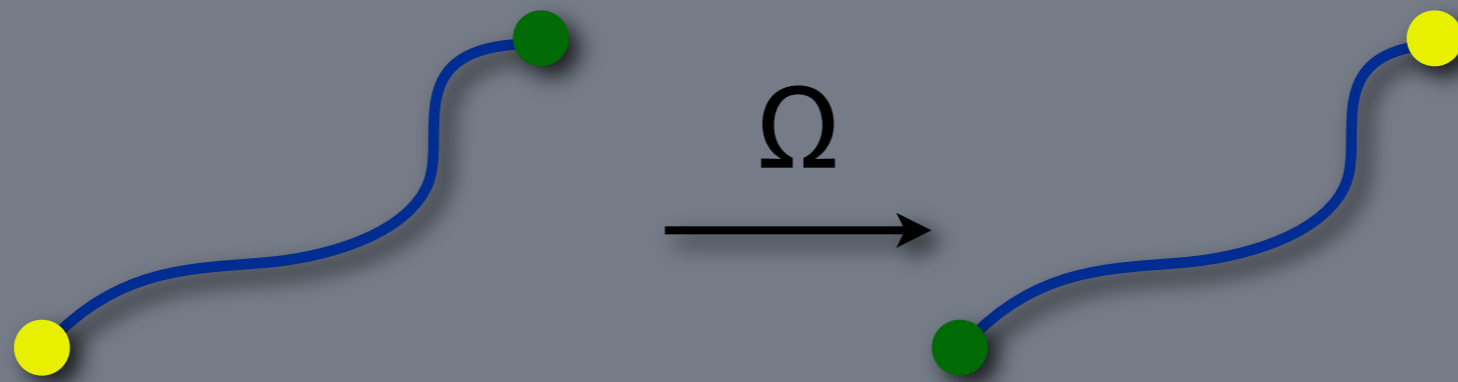
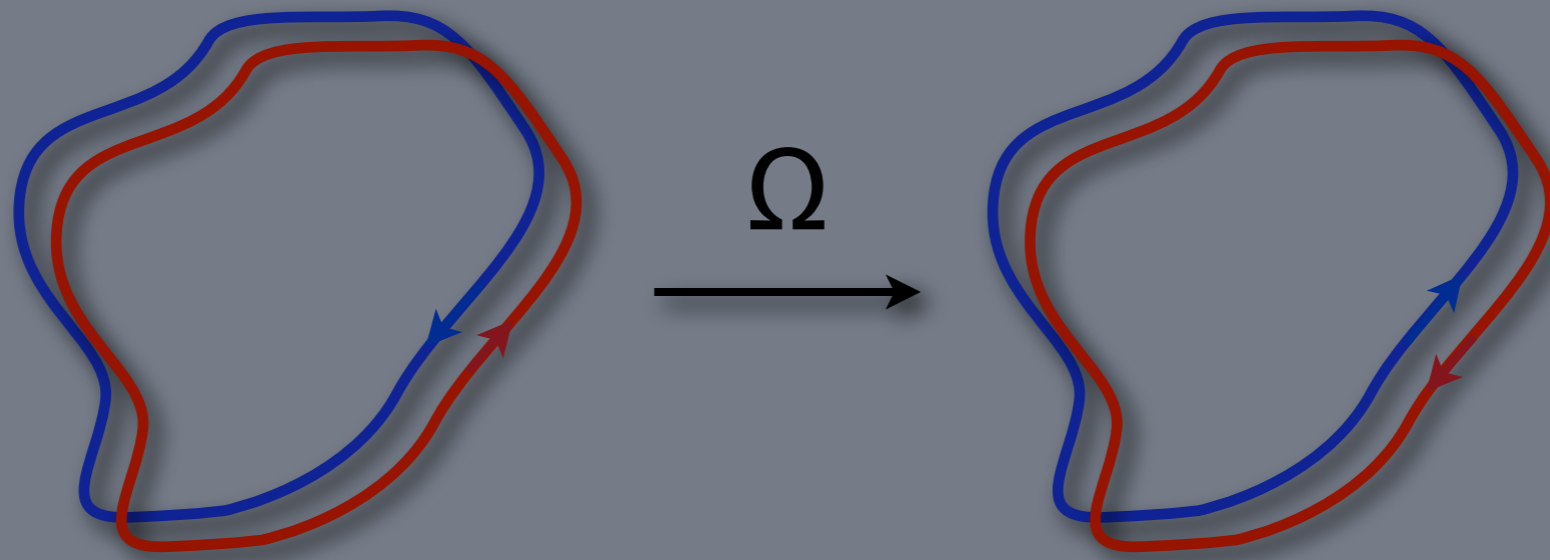
also: Govaerts 1989  
Horava 1989  
Bern, Dunbar 1990  
Green, 1994



Similarly to the GSO projection,  
arrange closed and open string states  
according to world-sheet parity  $\Omega$

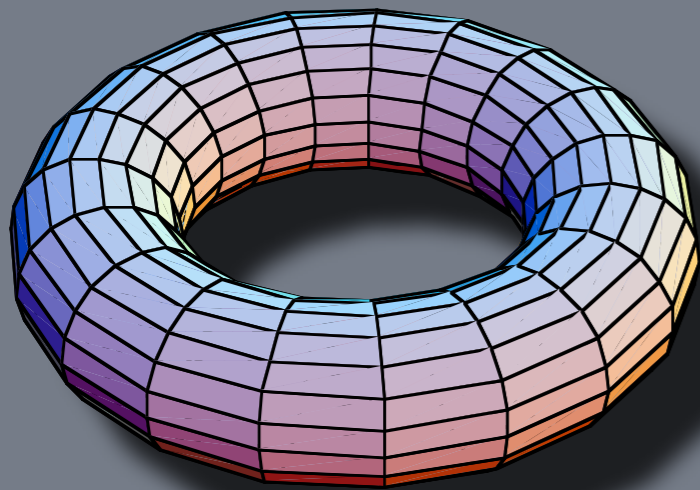
(this is allowed only if the parent theory is invariant under  $\Omega$ )

retain only the part of the spectrum  
with proper  $\Omega$  parity



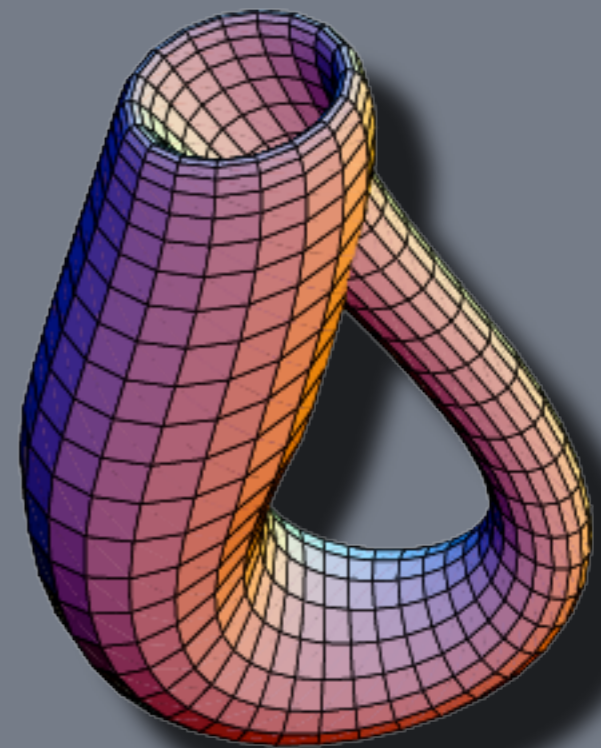
closed and open strings become unoriented!

# The closed-string sector



$$\mathcal{I} = \frac{1}{2} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^6} \frac{1}{|\eta|^{16}} (V_8 - S_8)(\bar{V}_8 - \bar{S}_8)$$

$$\mathcal{K} = \frac{1}{2} \int_0^\infty \frac{d\tau_2}{\tau_2^6} \frac{1}{\eta^8} (V_8 - S_8)(2i\tau_2)$$

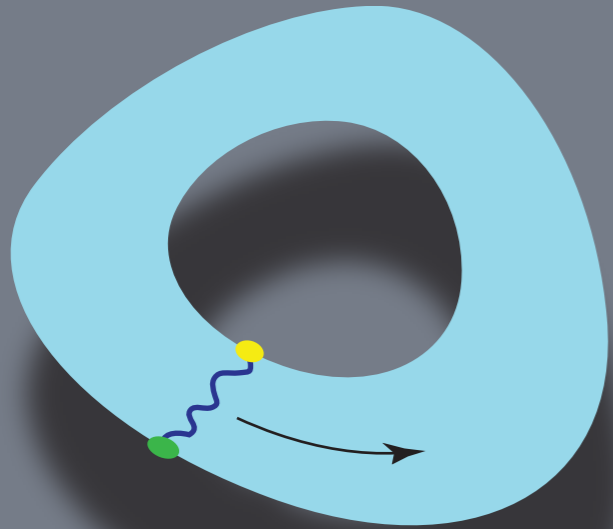


the massless closed-string excitations

$$\mathcal{T} + \mathcal{K} = g_{\mu\nu}, \phi, C_{\mu\nu}, \psi_{\text{L}}^{\mu}, \lambda_{\text{R}}$$

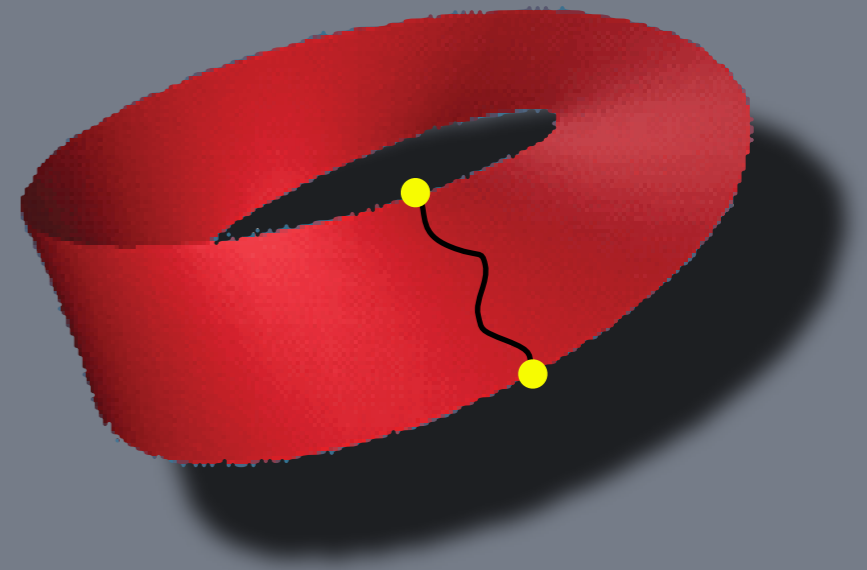
minimal D=10 N=(1,0) supergravity

# The open-string sector



$$\mathcal{A} = \frac{1}{2} N^2 \int_0^\infty \frac{dt}{t^6} \frac{1}{\eta^8} (V_8 - S_8) \left( \frac{it}{2} \right)$$

$$\mathcal{M} = \pm \frac{1}{2} N \int_0^\infty \frac{dt}{t^6} \frac{1}{\eta^8} (V_8 - S_8) \left( \frac{1+it}{2} \right)$$

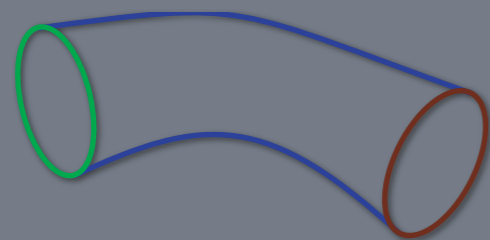


the massless open-string excitations

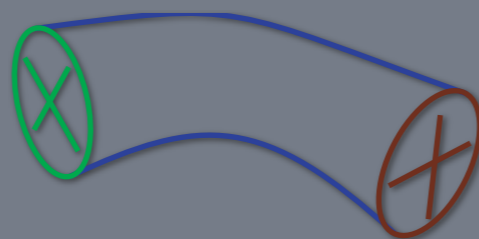
$$\mathcal{A} + \mathcal{M} = \frac{N^2 \pm N}{2} (A_\mu, \lambda_L)$$

minimal D=10 N=(1,0) super-Yang-Mills  
with gauge group SO(N) or USp(N)

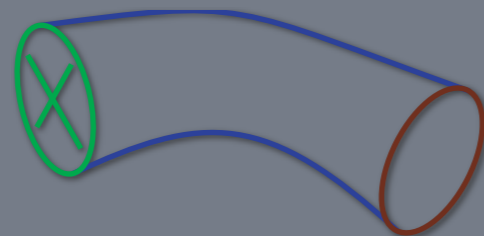
the new amplitudes are NOT modular invariant  
have a dual interpretation according to our choice of proper time



annulus

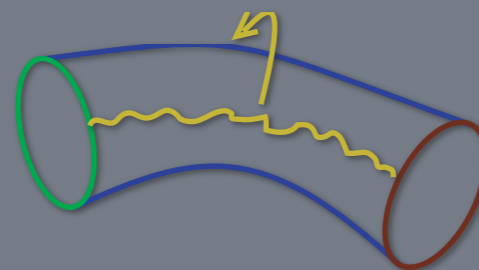


Klein bottle



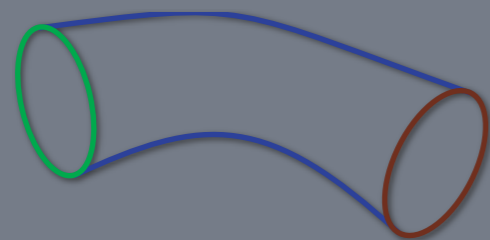
Möbius strip

for “vertical time”

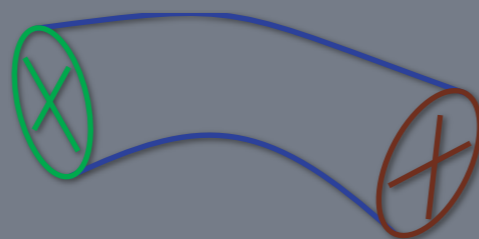


one loop for  
open strings

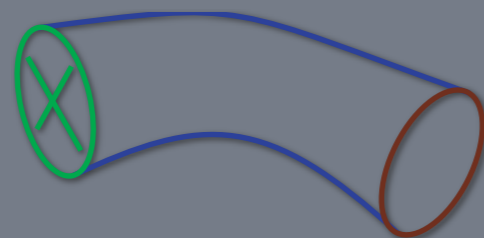
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annulus

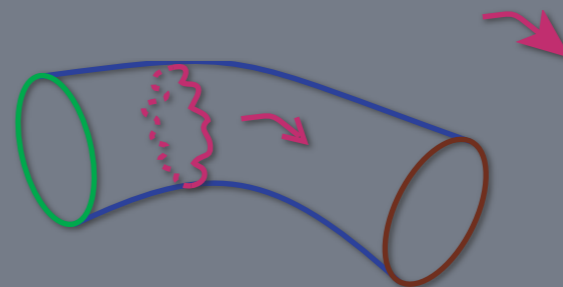


Klein bottle



Möbius strip

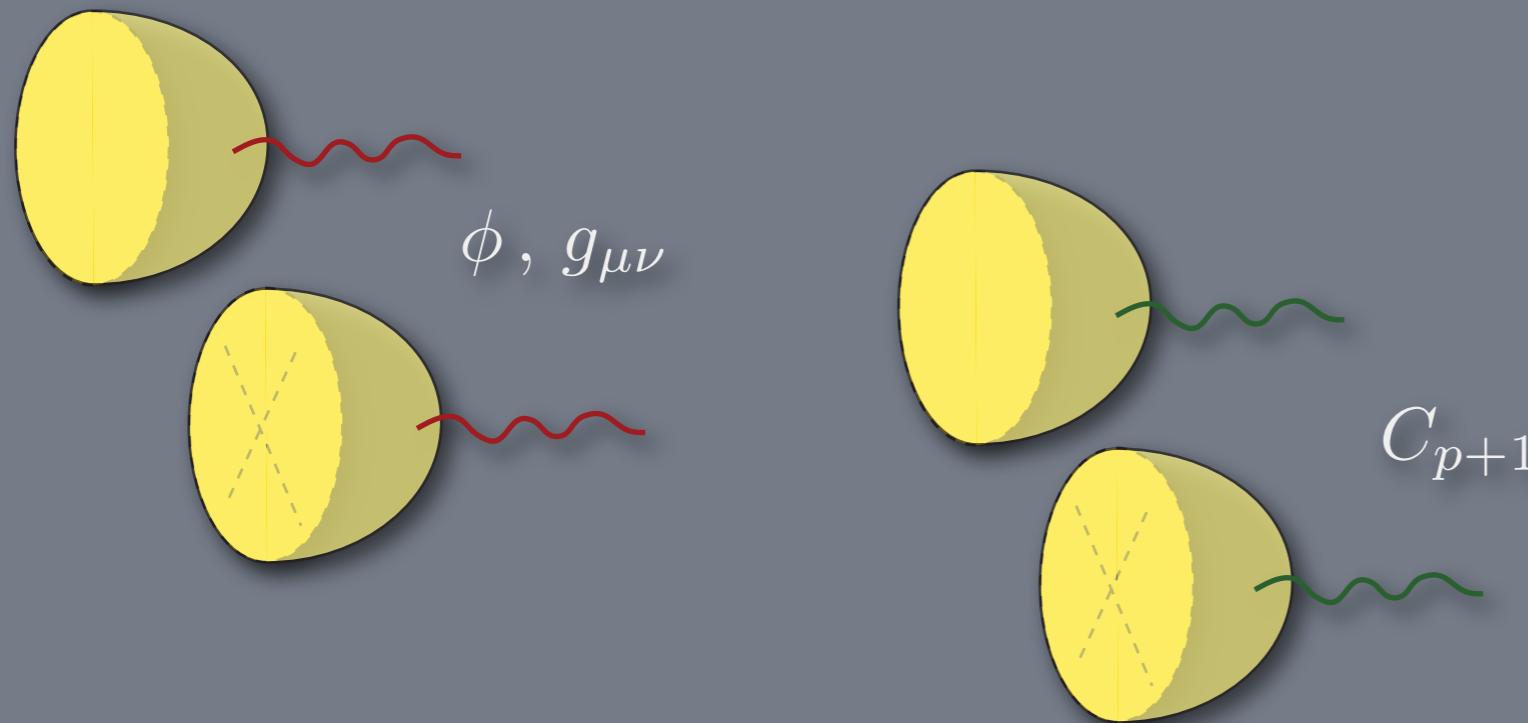
for "horizontal time"



tree level  
free propagation  
for closed strings



# ***divergencies are due to the exchange of closed-string massless states***



Green, Schwarz 1985  
Douglas, Grinstein 1987  
Marcus, Sagnotti 1987  
Weinberg 1987  
Bianchi, Sagnotti 1988

picture  $(-\frac{1}{2}, -\frac{3}{2})$

Bianchi, Pradisi, Sagnotti 1992

cancellation of tadpoles

$$N \pm 32 = 0$$

uniquely selects the gauge group

$SO(32)$

*in this framework the cancellation of RR tadpoles automatically gives anomaly-free theories*

from the point of view of low-energy supergravity  
the non-vanishing tadpoles introduce new couplings

$$(\tau_D + \tau_{\mathcal{O}}) \int_{\mathcal{M}_{p+1}} d^{p+1}x \sqrt{-g} e^{-\phi} + (q_D + q_{\mathcal{O}}) \int_{\mathcal{M}_{p+1}} C_{p+1}$$

these correspond to tension and RR charge  
of extended objects

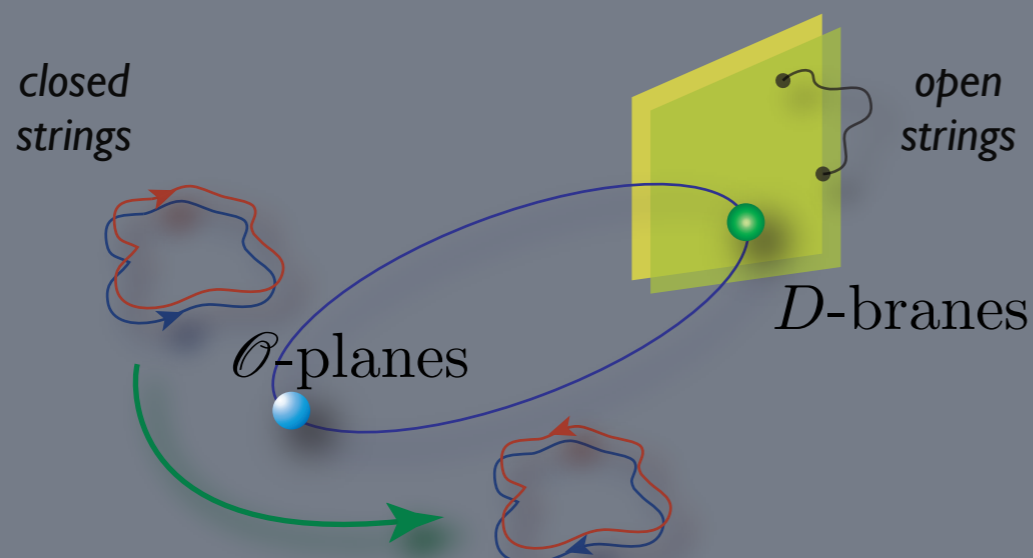
## Dirichlet Branes and Ramond-Ramond Charges

Joseph Polchinski\*

*Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030*

(Received 10 October 1995)

We show that D-branes, extended objects defined by mixed Dirichlet-Neumann boundary conditions, break half the supersymmetries of the type II superstring and carry a complete set of electric and magnetic Ramond-Ramond charges. The product of the electric and magnetic charges is a single Dirac unit, and the quantum of charge is that required by string duality. This is strong evidence that D-branes are intrinsic to type II string theory and are the Ramond-Ramond sources needed for string duality. Also, we find in the IIA string a 9-form potential, which gives an effective cosmological constant.



	$\tau$	$q$
$\mathcal{O}^+$	-	-
$\mathcal{O}^-$	+	+
$D$	+	+

Tadpole conditions:

$$\sum q_{\mathcal{O}} + q_D = 0 \quad (\text{neutrality})$$

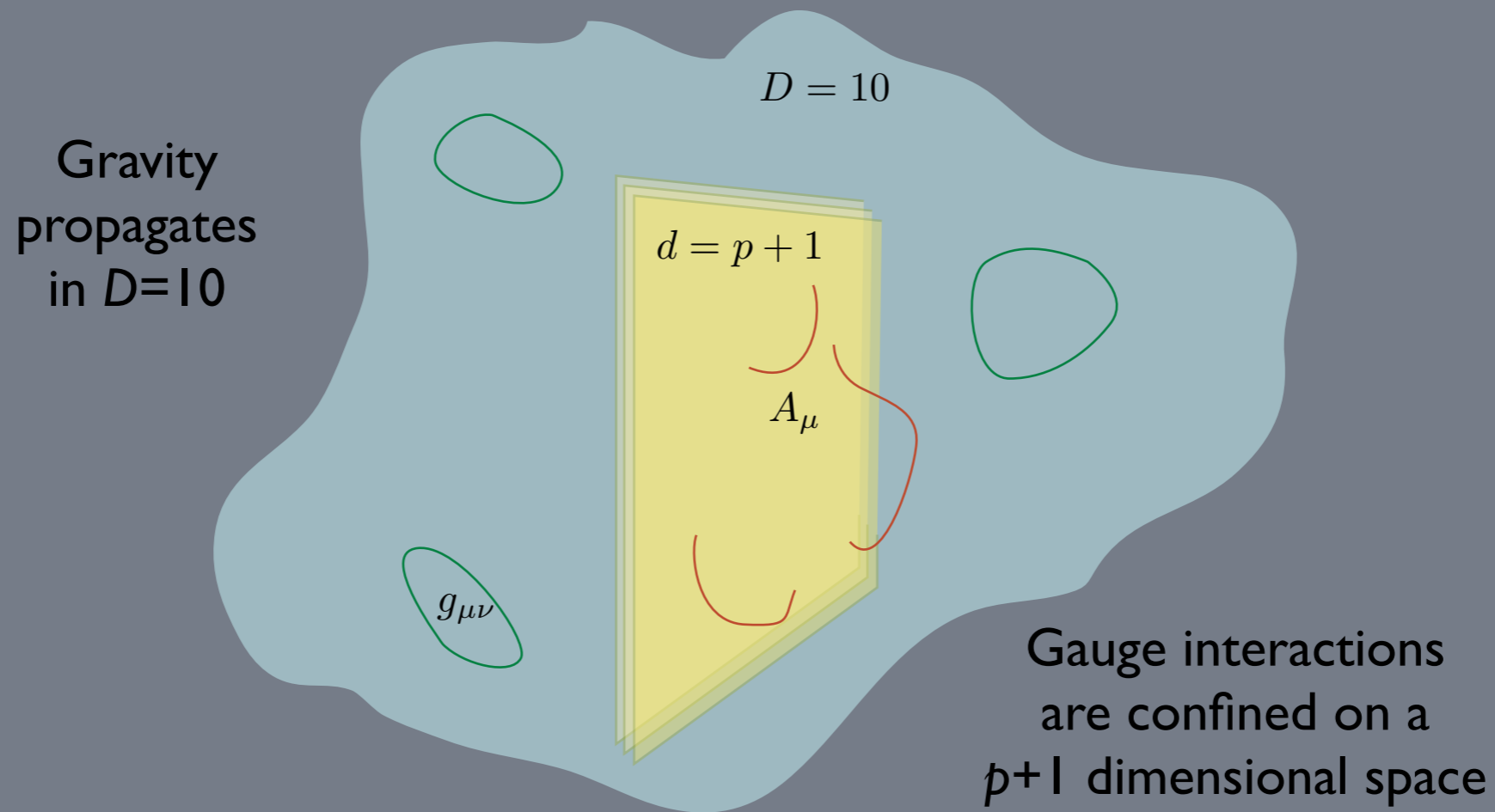
$$\sum \tau_{\mathcal{O}} + \tau_D = 0 \quad (\text{massless})$$

open-string  
invariant  $d.o.f.$

NS :  $\text{sign}(\tau_{\mathcal{O}}\tau_D)$  sym.

R :  $\text{sign}(q_{\mathcal{O}}q_D)$  sym.

# Gravitational vs gauge interactions



$$g_{\mu\nu} = g_{\mu\nu}(x^\mu, y^a)$$

$$A_\mu = A_\mu(x^\mu)$$

new scenarios for string phenomenology

(large extra dimensions, intersecting brane worlds, ...)

# ***an instance of string-string duality***

*minimal 10D supergravity is unique  
modulo the choice of the gauge group*

*heterotic  $SO(32)$*

*type I superstring*

*have different microscopic d.o.f.*

***are the two theories related in some way?***

# low-dimensional supergravity in the string frame

$$\int d^{10}x \sqrt{-g_H} e^{-2\phi_H} [R + (\partial\phi_H)^2 + (dB)^2 + F^2]$$

$$\int d^{10}x \sqrt{-g_I} [e^{-2\phi_I} (R + (\partial\phi_I)^2) + (dC)^2 + e^{-\phi_I} F^2]$$

field redefinition  $\phi_H = -\phi_I \Rightarrow g_{s,H} = \frac{1}{g_{s,I}}$

strong-weak coupling duality

Polchinski, Witten 1995  
see Hull's talk

# ***lower-dimensional compactifications***

Bianchi, Sagnotti 1992  
Gimon, Polchinski 1996

for instance  $T^4/\mathbb{Z}_2$

closed strings  $D=6$   $N=(1,0)$  supergravity coupled to  
 $1$  tensor multiplet and  $20$  hypers

open strings gauge group  $U(16)\times U(16)$   
hypers in  $2(120,1)+2(1,120)+(16,16)$

RR tadpole conditions ensure cancellation of  
irreducible anomalies

$$273 - 29n_T + n_V - n_H = 0$$

reducible anomalies factorise

$$\mathcal{I}_8 \sim (\text{tr}R^2 - \text{tr}F_1^2 - \text{tr}F_2^2) (\text{tr}F_1^2 + \text{tr}F_2^2)$$

can be disposed of by the Green-Schwarz mechanism

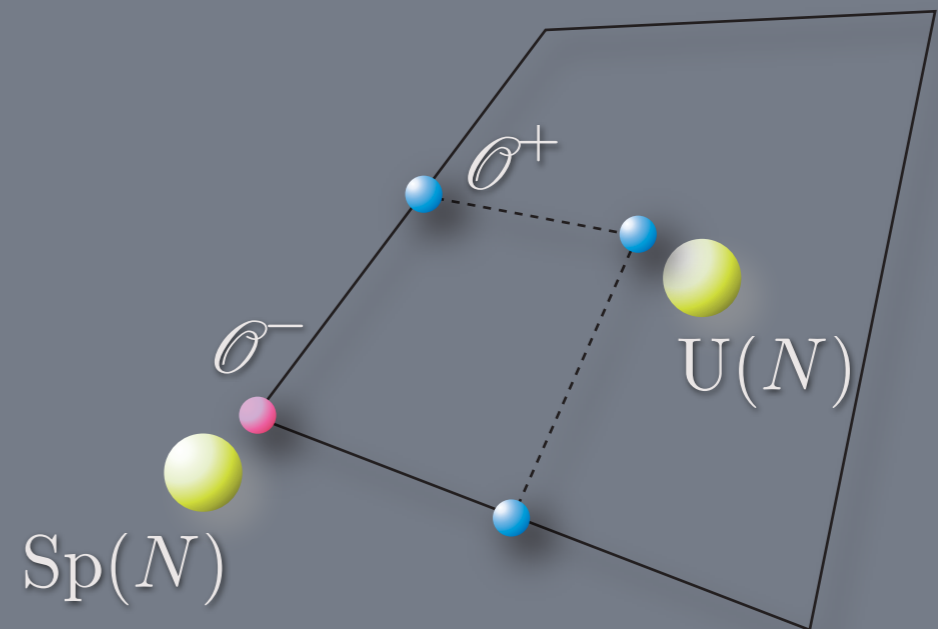


# $T^4/\mathbb{Z}_2$ with a non-vanishing $B_{ab}$ background

Bianchi, Pradisi, Sagnotti 1992  
C.A. 1999

new features:

- rank reduction of gauge group
- unitary and symplectic groups
- varying number of tensor multiplets



as usual, irreducible anomalies cancel  
if RR tadpoles conditions are imposed

reducible anomaly does not factorise any more

$$\begin{aligned}\mathcal{I}_8^{(r)} &= -\frac{2^{r/2}}{64} \left( 2^{(2-r)/2} \text{tr} R^2 - \text{tr} F_1^2 - \text{tr} F_2^2 - \text{tr} F_3^2 - \text{tr} F_4^2 \right)^2 + \\ &+ \frac{2^{r/2}}{64} \left( \text{tr} F_1^2 + \text{tr} F_2^2 - \text{tr} F_3^2 - \text{tr} F_4^2 \right)^2 + \\ &+ \frac{2^{r/2}}{64} \left( 2 - 2^{(r-2)/2} \right) \left( \text{tr} F_1^2 - \text{tr} F_2^2 + \text{tr} F_3^2 - \text{tr} F_4^2 \right)^2 + \\ &+ \frac{2^{r/2}}{64} \left( 4 - 2^{(r-2)/2} \right) \left( \text{tr} F_1^2 - \text{tr} F_2^2 - \text{tr} F_3^2 + \text{tr} F_4^2 \right)^2\end{aligned}$$

a generalised Green-Schwarz mechanism now takes place

*In general, for  $n_T$  tensor multiplets the residual anomaly polynomial takes the form*

Sagnotti 1992

Ferrara, Minasian, Sagnotti 1996

Ferrara, Riccioni, Sagnotti, 1998

$$\mathcal{I}_8 = - \sum_{x,y} c_x^r c_y^s \eta_{rs} \text{tr}_x F^2 \text{tr}_y F^2$$

*Accordingly, the field strengths of the 2-forms include combinations of Chern-Simons forms, and the kinetic term for the gauge vectors involves couplings to the scalars  $v_r$  in the tensor multiplets*

$$e^{-1} \mathcal{L} \sim -\frac{1}{2} v_r c^{rz} \text{tr}_z F_{\mu\nu} F^{\mu\nu}$$

*singularities  $\Rightarrow$  transitions to tensionless strings*

Seiberg, Witten 1996

Duff, Lu, Pope 1996

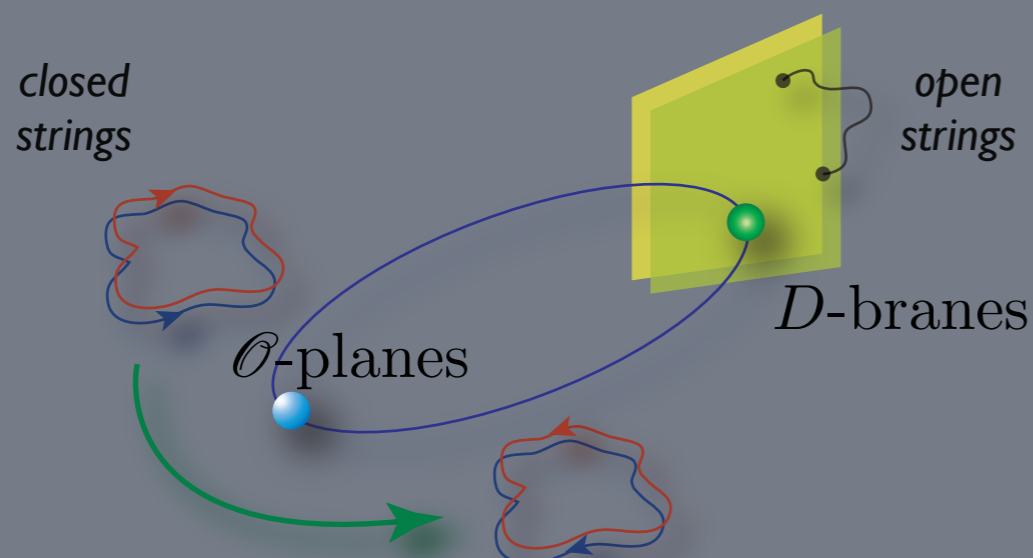
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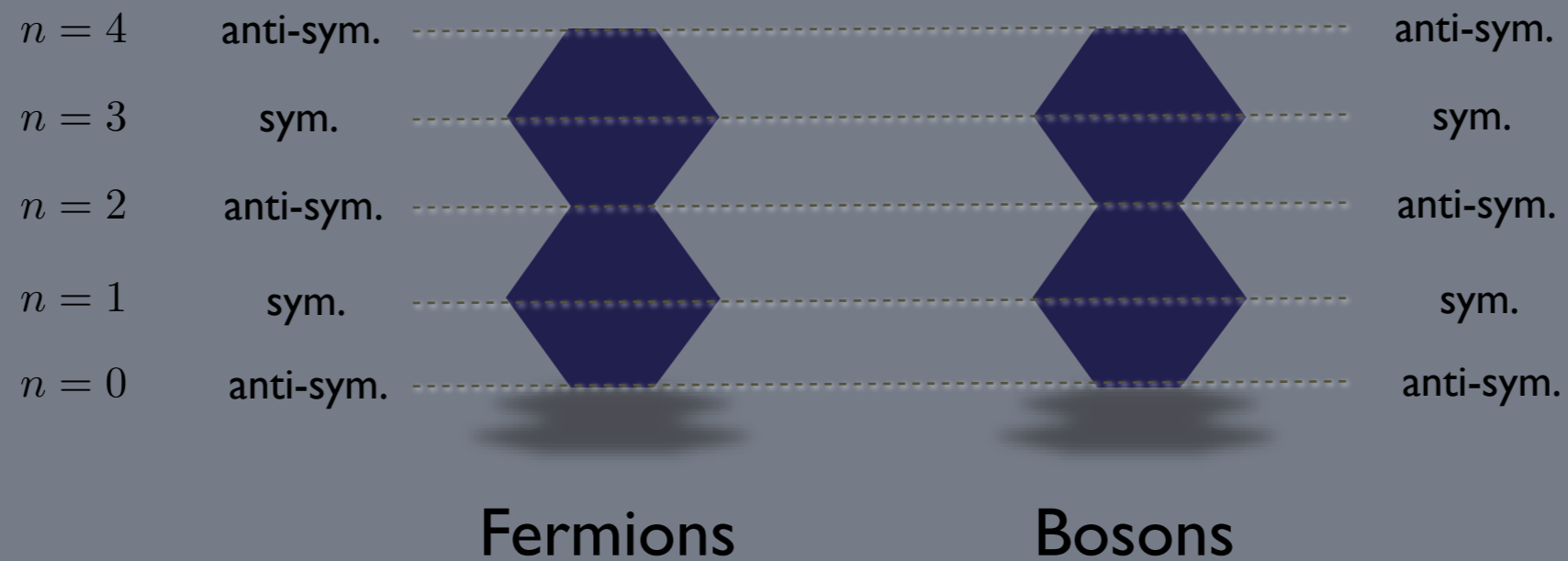
open-string  
invariant *d.o.f.*

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# Brane Supersymmetry

## Open-String spectrum

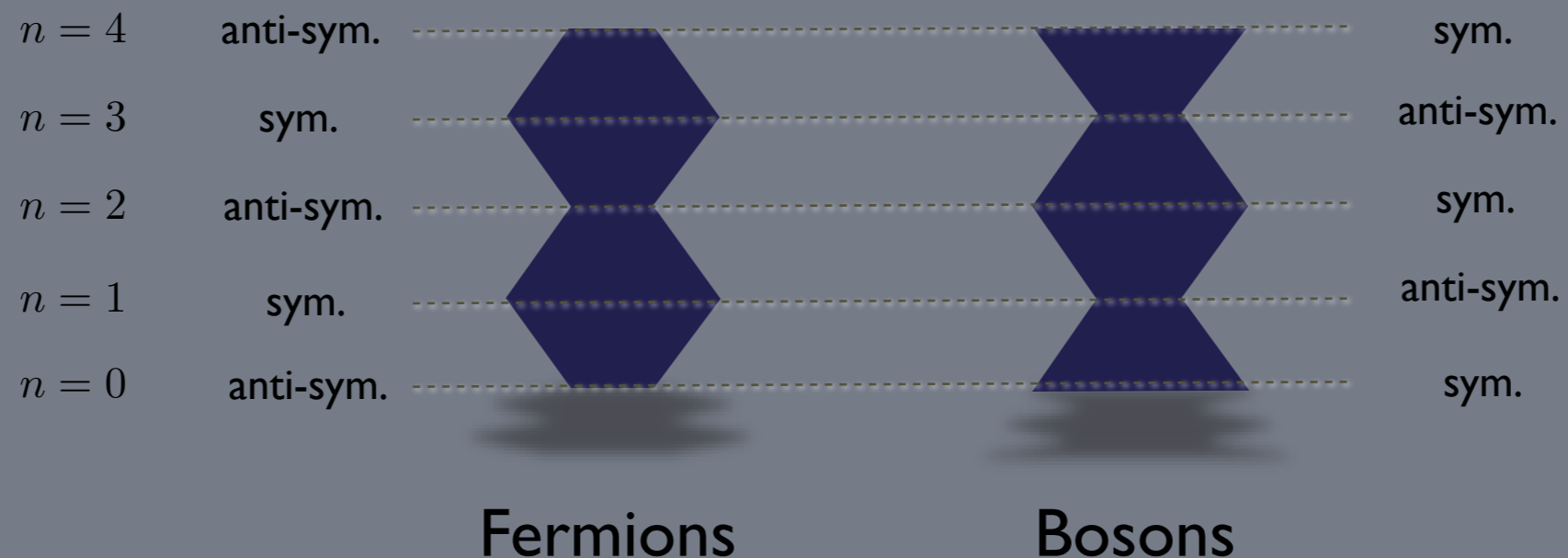


SO(32) and fermions in the adjoint

# Brane Supersymmetry *Breaking*

Sugimoto 1999  
 Antoniadis, Dudas, Sagnotti 1999  
 C.A. 1999  
 Aldazabal, Uranga 1999

## Open-String spectrum



USp(32) and fermions in the 496

$$\mathcal{O}^+ \oplus 16 \text{ D-branes} \quad \Rightarrow \quad \mathcal{O}^- \oplus 16 \text{ anti-D-branes}$$

$$\Delta M \sim 1/\sqrt{\alpha'} \quad \text{but} \quad \Lambda \quad \text{at tree-level}$$

*how to shift the vacuum in string theory?*

*how to implement the Fischler-Susskind mechanisms?*

***urgent problem for the younger generation!!***