

# Time-dependent AdS/CFT Correspondence: Applications

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(IPhT, Saclay)

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- (1/2) Lecture I: AdS/CFT and *Strong Gauge Interactions*  
*Two Facets of AdS/CFT applications*
- (1/2) Lecture II: AdS/CFT and *late time* QGP flow  
*Einstein/Hydro Duality*
- (1/2) Lecture III: AdS/CFT and *early time* QGP flow  
*Far-from-equilibrium Duality*
- (1/2) Lecture IV: AdS/CFT and the cosmic flow  
*Cosmology/Moving-Plasma Duality*

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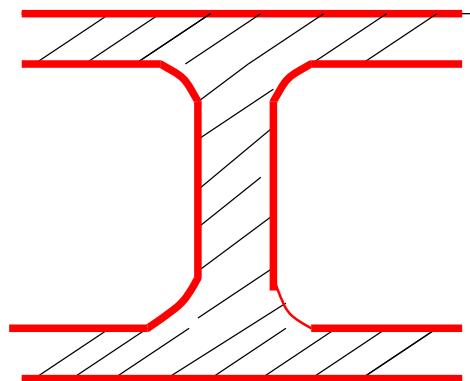
<sup>a</sup>Thanks to Romuald Janik and Philippe Brax

# Lecture I: AdS/CFT and *Strong Interactions*

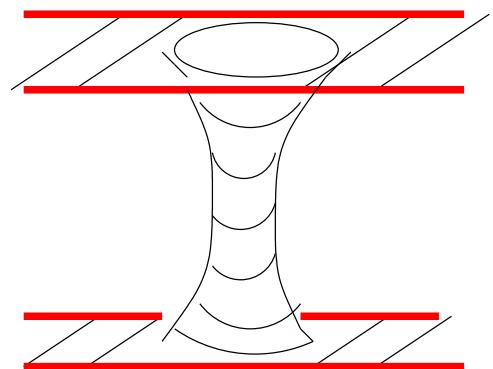
Strong Interactions and Strings: Historical Note  
Strings  $\not\leftrightarrow$  QCD  $\leftrightarrow$  Strings

1968  $\Rightarrow$  1974  $\Rightarrow$  1998  $\Rightarrow$  2008 ...

*Open String*



*Closed String*

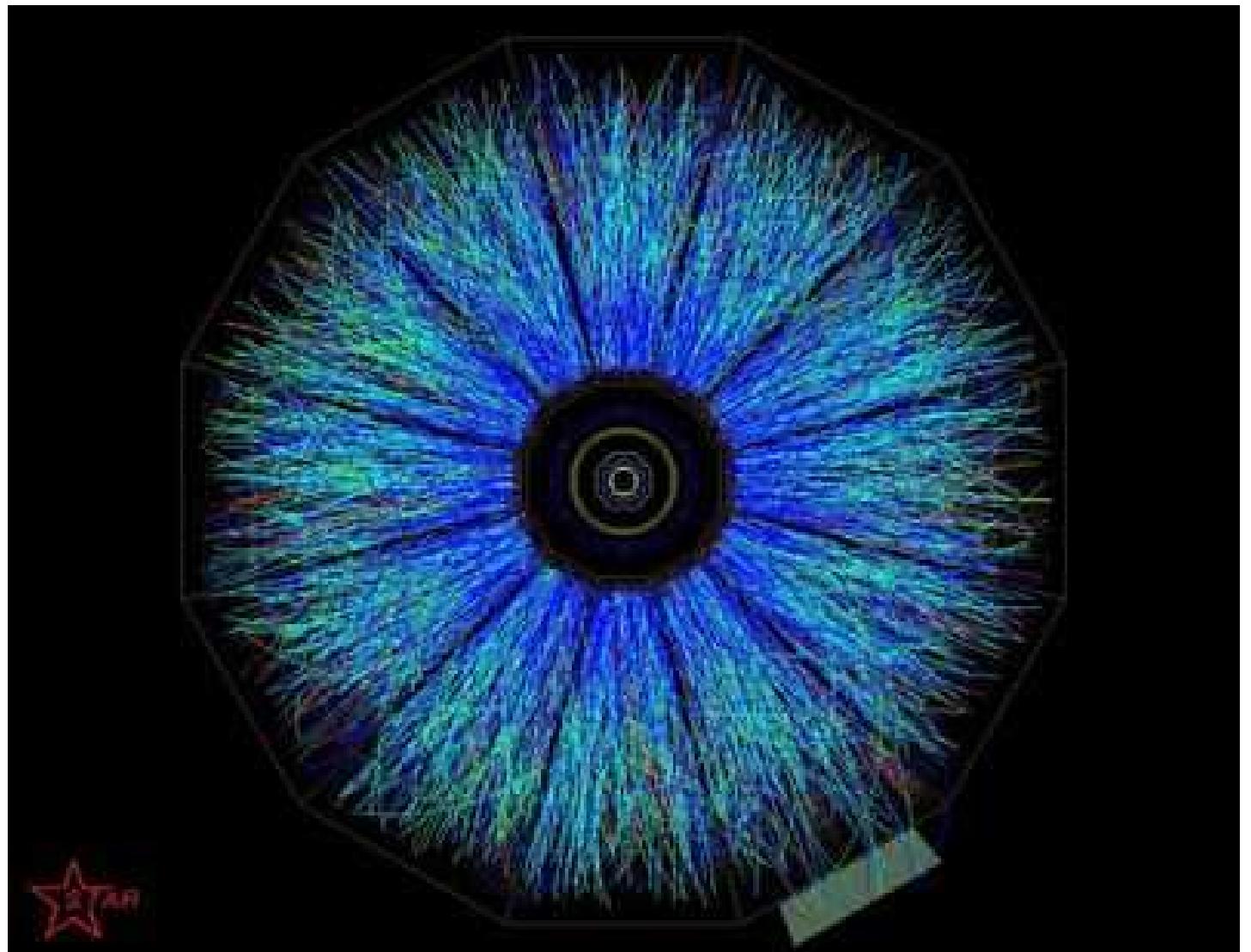


*Gauge*

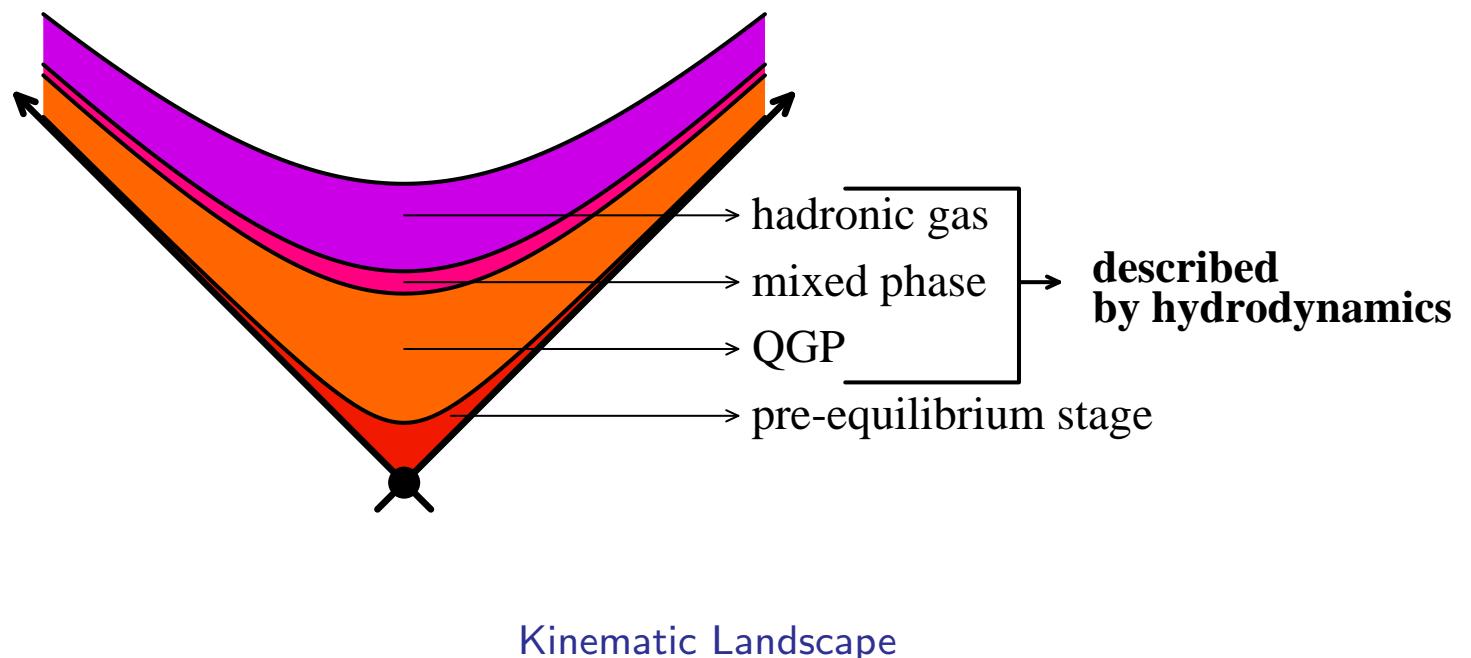
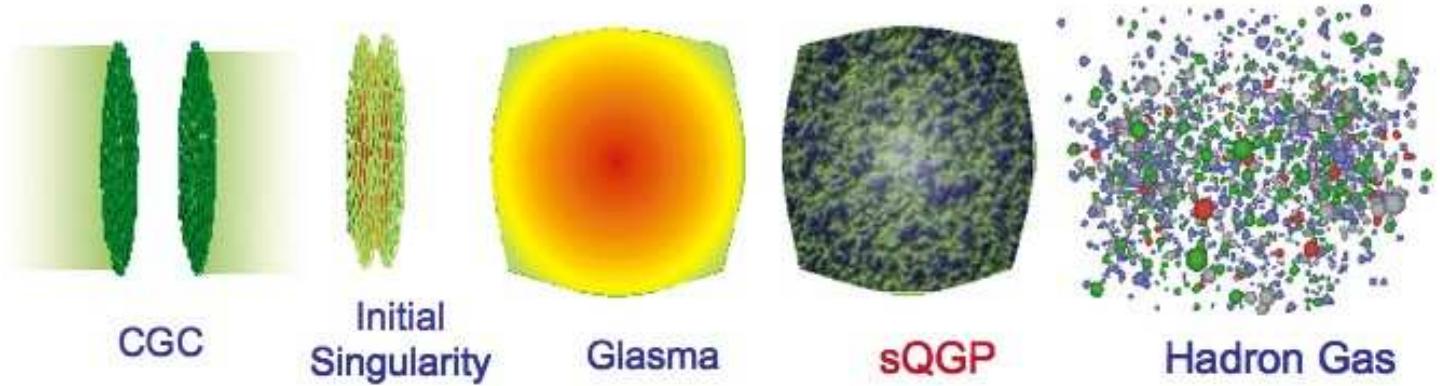
*Gravity*

Experimentally: A new light from heavy-ions

# Heavy-Ion Collision



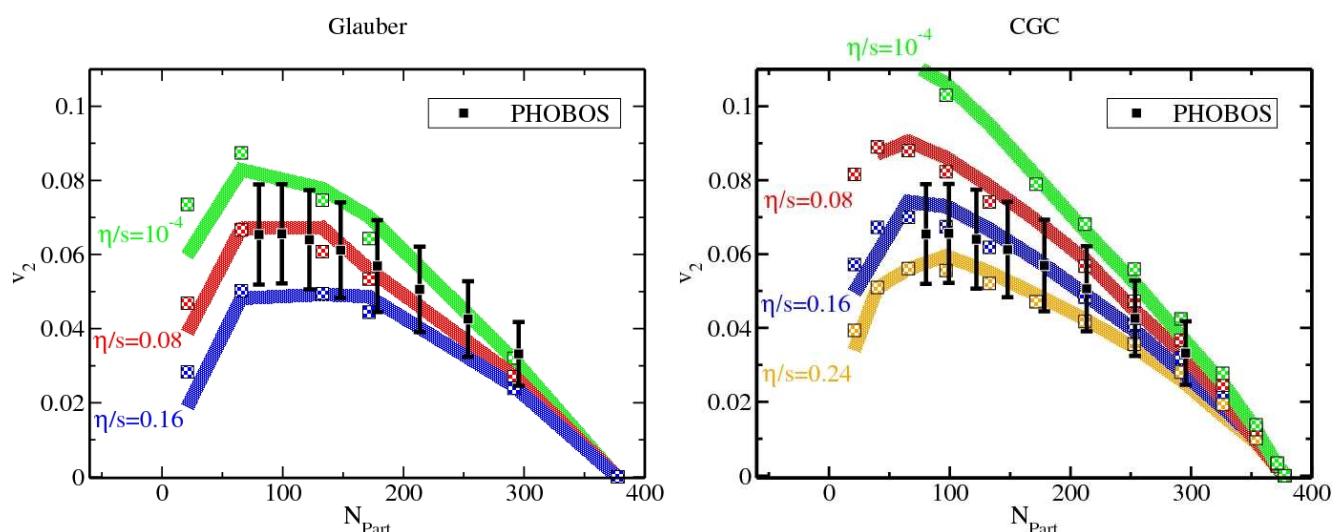
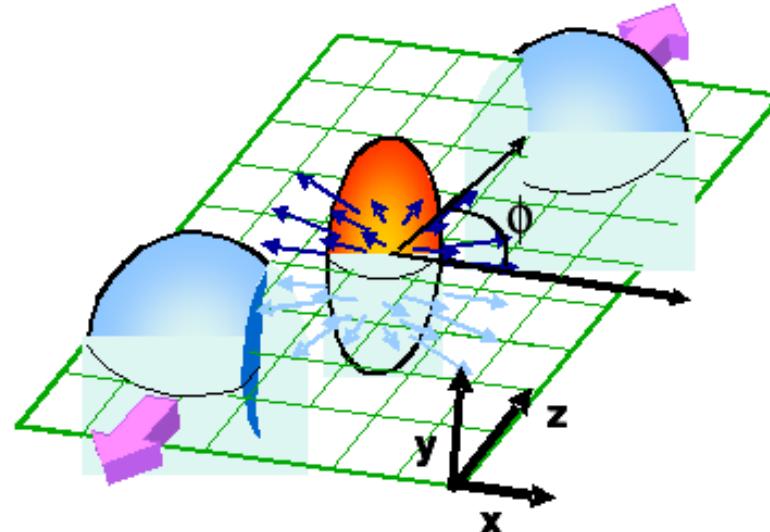
# QGP formation in heavy-ion collisions



$$\tau = \sqrt{x_0^2 - x_1^2} ; \quad \eta = \frac{1}{2} \log \frac{x_0 + x_1}{x_0 - x_1} ; \quad x_T = \{x_2, x_3\}$$

# Strings *vs.* Reality: Elliptic Flow

Ollitrault (1992)



Luzum, Romatschke (2008)

$$\frac{\partial N}{\partial \Phi} \propto 1 + 2 \, v_2 \cos 2\Phi$$

# Hydrodynamics of a perfect fluid

- Energy-Momentum Tensor

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - p\eta^{\mu\nu}$$

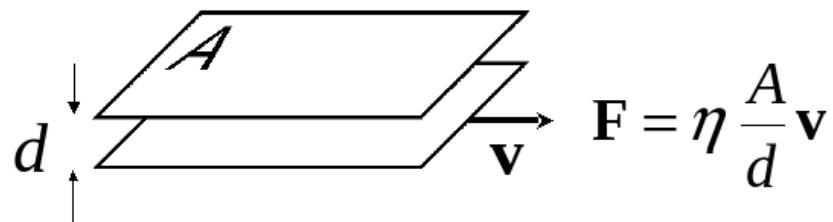
- Relativistic Hydrodynamic Equations (conformal)

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{EM Conservation condition}$$

$$T^{\mu\mu} = 0 : \quad \text{Traceless condition}$$

$$c_{sound}^2 \equiv \partial\epsilon/\partial p = 1/3 : \quad \text{Equation of State}$$

- Viscosity

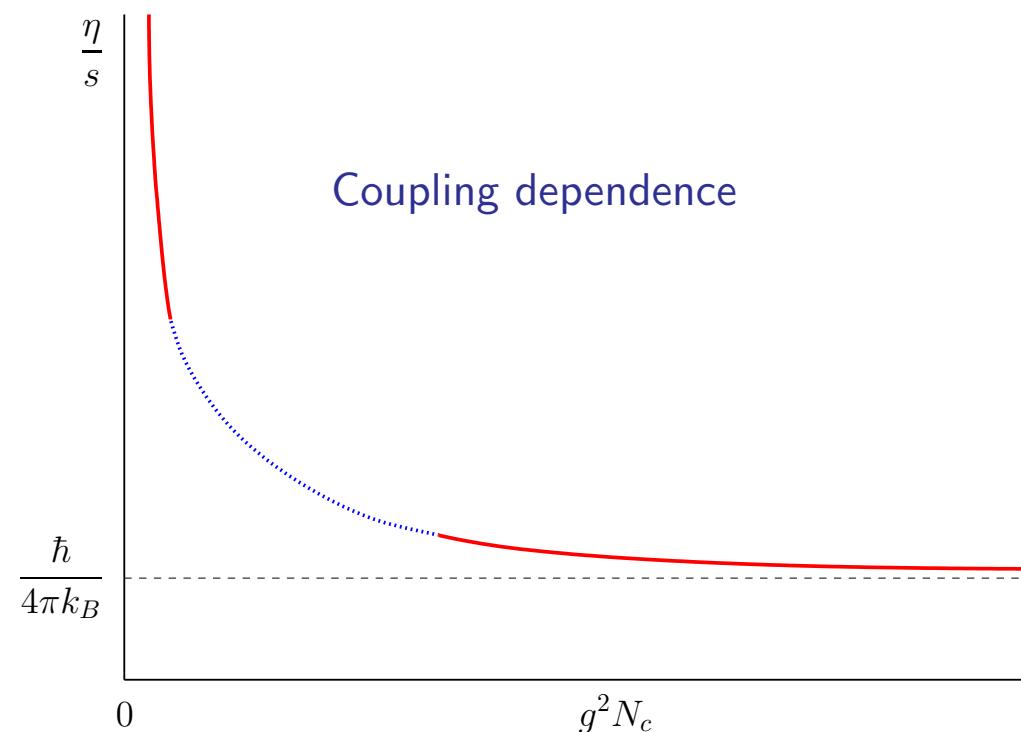
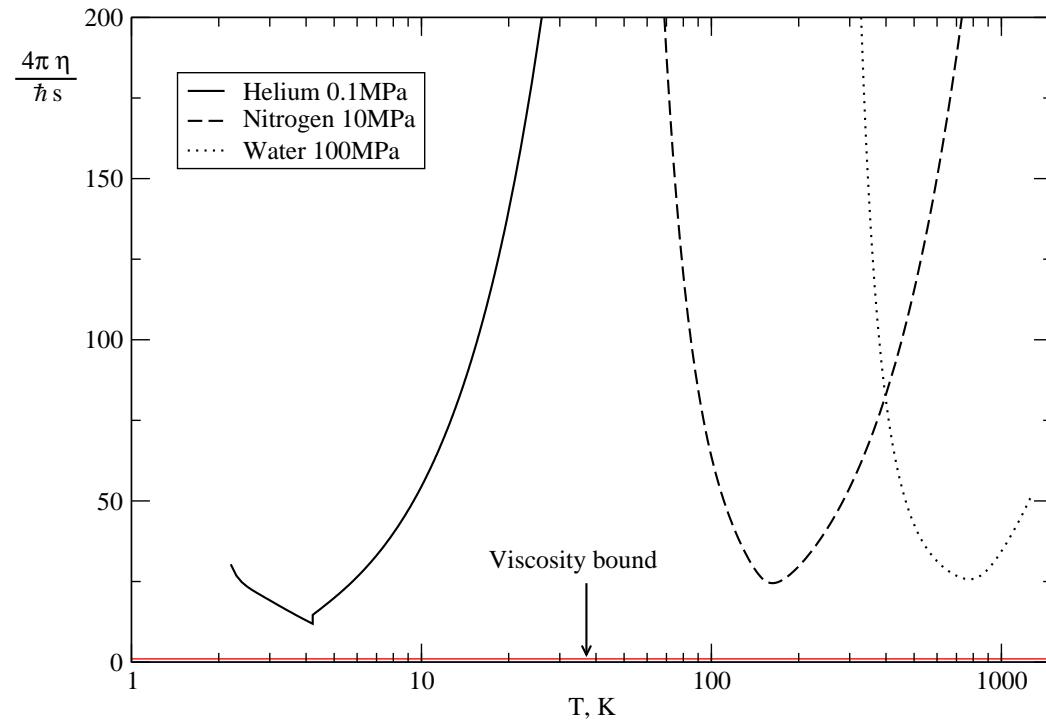


$$\frac{\eta}{s} = \frac{\rho_{part} \times \lambda_{free} \times \bar{p}_{av}}{\rho_{part} \times k_B} \geq \frac{1}{4\pi} \frac{\hbar}{k_B} = \frac{\eta}{s} \Big|_{AdS/CFT}$$

Kovtun, Son, Starinets (2004)

# Viscosity Bound

Kovtun, Son, Starinets



# QGP/BH Duality and Viscosity

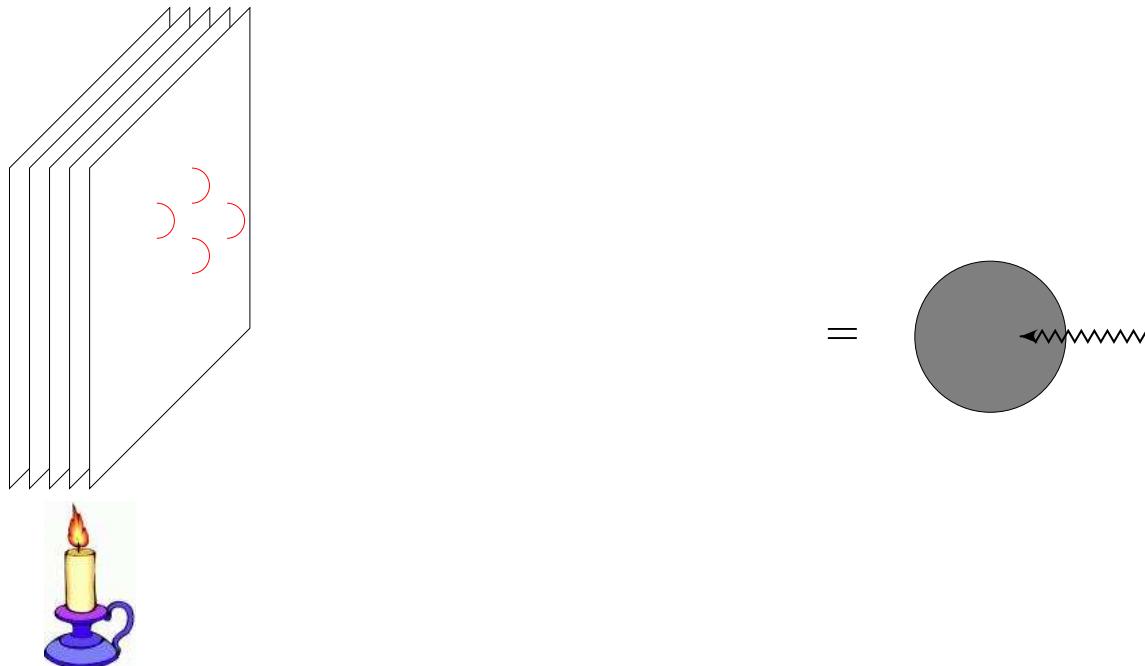
Static Case: PolICASTRO, Son, Starinets (2001)

## Viscosity on the light of duality

Consider a graviton that falls on this stack of  $N$  D3-branes

Will be absorbed by the D3 branes.

The process of absorption can be looked at from two different perspectives:



Absorption by D3 branes ( $\sim$  viscosity) = absorption by black hole

$$\sigma_{abs}(\omega) \propto \int d^4x \frac{e^{i\omega t}}{\omega} \langle [T_{x_2 x_3}(x), T_{x_2 x_3}(0)] \rangle \Rightarrow \frac{\eta}{s} \equiv \frac{\sigma_{abs}(0)/(16\pi G)}{A/(4G)} = \frac{1}{4\pi}$$

AdS/CFT correspondence and the Quark-Gluon Plasma

# From Experiments to Theory

Abstracted from RHIC Data :

- Evidence for an Hydrodynamic Flow
- QGP: (Almost) Perfect fluid behaviour  $\Rightarrow$  small viscosity
- Fast QGP Formation  $\Rightarrow$  fast thermalisation/isotropization
- “in-Medium” properties: flavors, meson spectrum, etc...

Interest of AdS/CFT :

- QGP as a Deconfined “Quasi-Conformal” phase  $\Rightarrow$   $N^4$ QCD useful
- AdS/CFT as a “laboratory” for QCD
- Gauge/Gravity: a wider concept

QGP at Strong Coupling : What Gauge/Gravity can tell us?

# Gauge/Gravity Duality and QGP Dynamics

Janik, R.P.

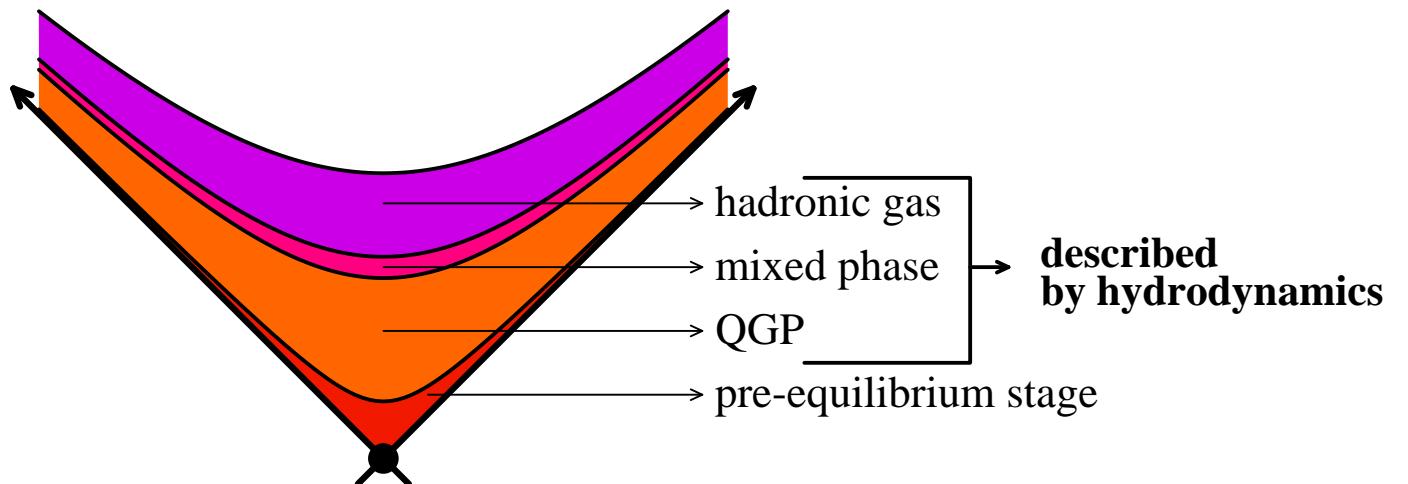
Janik, Heller, Benincasa, Buchel...

Kovchegov, Taliotis, Albacete,...

Nakamura, Sin, Kinoshita, Mukoyama, Nakamura, Oda, Natsuume, Okamura,...

Bhattacharyya, Hubeny, Minwalla, Ranganami, Loganayagam,...

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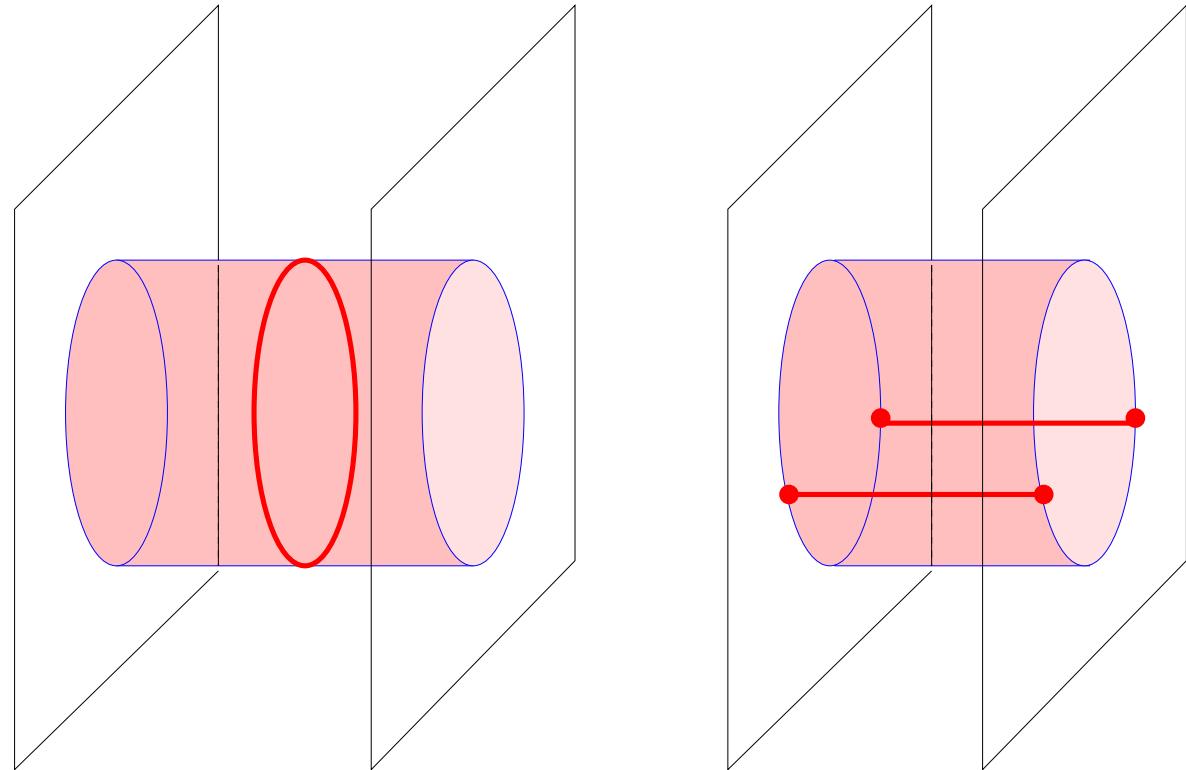
$$\tau = \sqrt{x_0^2 - x_1^2} ; \eta = \frac{1}{2} \log \frac{x_0 + x_1}{x_0 - x_1} ; x_T = x_2, x_3$$

## Questions

- What is the Gravity Dual of a Flow?
- QGP: (almost) Perfect fluid behaviour, why?
- Same  $\frac{\eta}{s}$ ?, Transport Coefficients, Navier-Stokes,...
- Fast Pre-equilibrium stage, why? see lecture III

# The Gauge-Gravity Correspondence

Open  $\Leftrightarrow$  Closed String duality

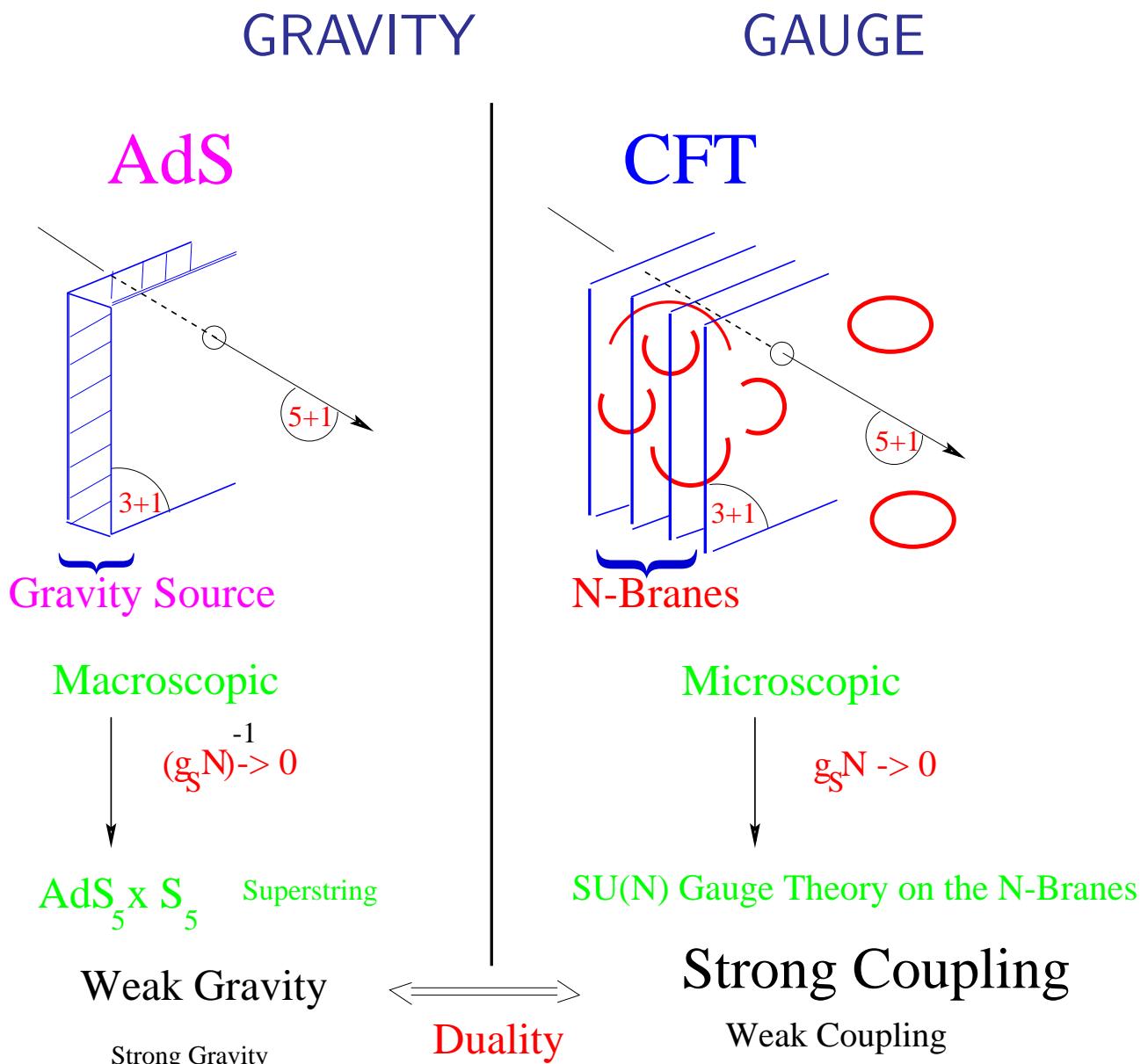


Schomerus

- |                             |                   |                                     |
|-----------------------------|-------------------|-------------------------------------|
| <i>Closed String</i>        | $\Leftrightarrow$ | <i>1-loop Open String</i>           |
| <i>Gravity</i>              | $\Leftrightarrow$ | <i>Gauge</i>                        |
| <i>D-Brane “Universe”</i>   | $\Rightarrow$     | <i>Open String Ending</i>           |
| <i>Small/Large Distance</i> | $\Rightarrow$     | <i>Gauge/Gravity Correspondence</i> |

# AdS/CFT Correspondence

J.Maldacena (1998)



# Holography at work

Brane → Bulk: Holographic Renormalization

K.Skenderis (2002)

- Bulk metric

$$ds^2 = \frac{g_{\mu\nu}(z) dx^\mu dx^\nu + dz^2}{z^2}$$

(in Fefferman-Graham Coordinates)

- For  $5d \Rightarrow 4d$  Minkowski metric:

$$g_{\mu\nu}(z) = g_{\mu\nu}^{(0)} (= \eta_{\mu\nu}) + z^2 g_{\mu\nu}^{(2)} (= 0) + z^4 g_{\mu\nu}^{(4)} (= \langle T_{\mu\nu} \rangle) + \mathbf{z}^6 \dots +$$

+  $\mathbf{z}^6 \dots +$ : from Einstein Eqs.

- Energy-Momentum Tensor (in general):

$$\langle T_{\mu\nu} \rangle = \frac{2l^3}{\kappa_5^2} \left\{ g_{\mu\nu}^{(4)} - \frac{1}{8} g_{\mu\nu}^{(0)} \left[ (\text{Tr}(g^{(2)}))^2 - (\text{Tr}g^{(2)})^2 \right] - \frac{1}{2} g_{\mu\rho}^{(2)} g^{(0)\rho\sigma} g_{\sigma\nu}^{(2)} + \frac{1}{4} (\text{Tr}g^{(2)}) g_{\mu\nu}^{(2)} \right\}$$

# An Illustration of Holographic Renormalization (static case)

5d Black Hole  $\Rightarrow$  4d Perfect Fluid

Balasubramanian,de Boer,Minic; Myers

- Exercise: 4d Perfect Fluid  $\Rightarrow$  5d Black Hole

Janik,R.P.

- Starting point

$$\langle T_{\mu\nu} \rangle \propto g_{\mu\nu}^{(4)} = \begin{pmatrix} 3/z_0^4 = \epsilon & 0 & 0 & 0 \\ 0 & 1/z_0^4 = p_1 & 0 & 0 \\ 0 & 0 & 1/z_0^4 = p_2 & 0 \\ 0 & 0 & 0 & 1/z_0^4 = p_3 \end{pmatrix}$$

- Resummation of Holographic Coefficients

$$g_{\mu\nu}(z) = g_{\mu\nu}^{(0)} (= \eta_{\mu\nu}) + z^2 g_{\mu\nu}^{(2)} (= 0) + z^4 g_{\mu\nu}^{(4)} (= \langle T_{\mu\nu} \rangle) + \text{z}^6 \dots +$$

- Derivation of the Fefferman-Graham metrics

$$ds^2 = -\frac{(1-z^4/z_0^4)^2}{(1+z^4/z_0^4)z^2} dt^2 + (1+z^4/z_0^4) \frac{dx^2}{z^2} + \frac{dz^2}{z^2}$$

- $z \rightarrow \tilde{z} = z \times \left\{ 1 + \frac{z^4}{z_0^4} \right\}^{-1/2}$ : 5-d Black Brane with horizon at  $\tilde{z}_0$

$$ds^2 = -\frac{1-\tilde{z}^4/\tilde{z}_0^4}{\tilde{z}^2} dt^2 + \frac{dx^2}{\tilde{z}^2} + \frac{1}{1-\tilde{z}^4/\tilde{z}_0^4} \frac{d\tilde{z}^2}{\tilde{z}^2}$$

## Lecture II: AdS/CFT and *late time* QGP flow

- Energy-Momentum Tensor: Perfect Fluid

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - p\eta^{\mu\nu}$$

- Relativistic Hydrodynamic Equations

$$\partial_\mu T^{\mu\nu} = 0 : \quad \text{Continuity condition}$$

$$T^{\mu\mu} = 0 : \quad \text{Traceless condition}$$

$$\epsilon = pc_{sound}^{-2} : \quad \text{Equation of State}$$

- Thermodynamical Identities

$$p + \epsilon = \epsilon (1 + c_s^2) = TS ; \quad d\epsilon = TdS$$

- Conformal metric

$$\epsilon = 3p = \epsilon_0 T^4 ; \quad S = S_0 T^3 \propto \epsilon^{3/4}$$

- "Bjorken Flow"

$$\epsilon(\tau) = 3p(\tau) = \epsilon_0 T^4(\tau)$$

No  $\eta$ -dependence (Central rapidity)

## AdS/CFT implies Perfect Fluid at late $\tau$

- Family of Boost-invariant  $T_\nu^\mu$

$$T_{\mu\nu} = \begin{pmatrix} f(\tau) & 0 & 0 & 0 \\ 0 & -\tau^3 \frac{d}{d\tau} f(\tau) - \tau^2 f(\tau) & 0 & 0 \\ 0 & 0 & f(\tau) + \frac{1}{2}\tau \frac{d}{d\tau} f(\tau) & 0 \\ 0 & 0 & 0 & \dots \end{pmatrix}$$

- Proper-time evolution

$f(\tau) \propto \tau^{-s}$

$$T_{\mu\nu} t^\mu t^\nu \geq 0 \Rightarrow 0 < s < 4 : \text{ Positivity of E-M.T.}$$

$f(\tau) \propto \tau^{-\frac{4}{3}}$  : Perfect Fluid  $\epsilon = 3p_\perp = 3p_L$  ("hydro")

$f(\tau) \propto \tau^{-1}$  : Free streaming  $\epsilon = 2p_\perp$ ;  $p_L = 0$  ("classical")

$f(\tau) \propto \tau^{-0}$  : Full Anisotropy  $\epsilon = p_\perp = -p_L$  ("early times?")

# Expanding Geometries

- General Boost-Invariant F-G metric:

$$ds^2 = \frac{-e^{a(\tau,z)} d\tau^2 + \tau^2 e^{b(\tau,z)} dy^2 + e^{c(\tau,z)} dx_\perp^2}{z^2} + \frac{dz^2}{z^2}$$

- Einstein Equation(s):

$$\{a(\tau, z), b(\tau, z), c(\tau, z)\} = \{a(v), b(v), c(v)\} + \mathcal{O}\left(\frac{1}{\tau^\#}\right)$$

$$\boxed{\text{Asymptotic Scaling} \quad \Rightarrow \quad v = \frac{z}{\tau^{s/4}}}$$

$$\begin{aligned} v(2a'(v)c'(v) + a'(v)b'(v) + 2b'(v)c'(v)) - 6a'(v) - 6b'(v) - 12c'(v) + vc'(v)^2 &= 0 \\ 3vc'(v)^2 + vb'(v)^2 + 2vb''(v) + 4vc''(v) - 6b'(v) - 12c'(v) + 2vb'(v)c'(v) &= 0 \\ 2vsb''(v) + 2sb'(v) + 8a'(v) - vsa'(v)b'(v) - 8b'(v) + vsb'(v)^2 + \\ 4vsc''(v) + 4sc'(v) - 2vs a'(v)c'(v) + 2vsc'(v)^2 &= 0 . \end{aligned}$$

- Asymptotic Solution

$$\begin{aligned} a(v) &= A(v) - 2m(v) \\ b(v) &= A(v) + (2s - 2)m(v) \\ c(v) &= A(v) + (2 - s)m(v) \end{aligned}$$

$$A(v) = \frac{1}{2} \left( \log(1 + \Delta(s) v^4) + \log(1 - \Delta(s) v^4) \right) \quad m(v) = \frac{1}{4\Delta(s)} \left( \log(1 + \Delta(s) v^4) - \log(1 - \Delta(s) v^4) \right) \quad \Delta(s) = \sqrt{(3s^2 - 8s + 8)/24}$$

# General Scaling Solution

$$v = \frac{z}{\tau^{1/4}}$$

- Asymptotic metric

$$\begin{aligned} z^2 \ ds^2 = & -(1 + \frac{v^4}{\sqrt{8}})^{\frac{1-2\sqrt{2}}{2}} (1 - \frac{v^4}{\sqrt{8}})^{\frac{1+2\sqrt{2}}{2}} dt^2 + (1 + \frac{v^4}{\sqrt{8}})^{\frac{1}{2}} (1 - \frac{v^4}{\sqrt{8}})^{\frac{1}{2}} \tau^2 dy^2 + \\ & + (1 + \frac{v^4}{\sqrt{8}})^{\frac{1+\sqrt{2}}{2}} (1 - \frac{v^4}{\sqrt{8}})^{\frac{1-\sqrt{2}}{2}} dx_\perp^2 + dz^2 \end{aligned}$$

- Investigating the Geometry:

Ricci scalar:

$$R = -20 + \mathcal{O}\left(\frac{1}{\tau^\#}\right)$$

Riemann tensor squared:

$$\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}$$

# Curvature invariant $\mathfrak{R}^2$

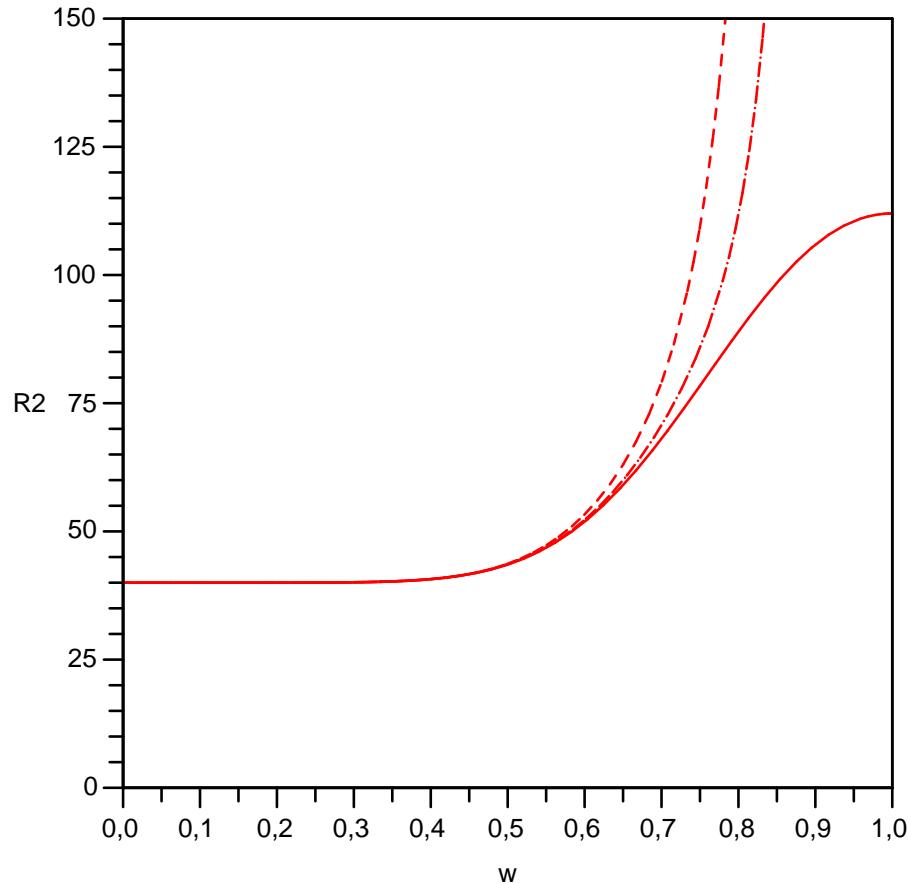
$$\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}$$

**Generic Solution:**

$$\begin{aligned}\mathfrak{R}^2 = & \frac{4}{\left(1 - \Delta(s)^2 v^8\right)^4} \times \left[ 10 \Delta(s)^8 v^{32} - 88 \Delta(s)^6 v^{24} + 42 v^{24} s^2 \Delta(s)^4 + \right. \\ & + 112 v^{24} \Delta(s)^4 - 112 v^{24} \Delta(s)^4 s + 36 v^{20} s^3 \Delta(s)^2 - 72 v^{20} s^2 \Delta(s)^2 + \\ & + 828 \Delta(s)^4 v^{16} + 288 v^{16} \Delta(s)^2 s - 288 v^{16} \Delta(s)^2 - 108 v^{16} s^2 \Delta(s)^2 + \\ & - 136 v^{16} s^3 + 27 v^{16} s^4 - 320 v^{16} s + 160 v^{16} + 296 v^{16} s^2 + 36 v^{12} s^3 + \\ & \left. - 72 v^{12} s^2 - 88 \Delta(s)^2 v^8 + 42 v^8 s^2 + 112 v^8 - 112 v^8 s + 10 \right] + \mathcal{O}\left(\frac{1}{\tau^\#}\right)\end{aligned}$$

# AdS/CFT: Selection of the Perfect Fluid

Singular Scalar  $\mathfrak{R}^2$  for  $s = \frac{4}{3} \pm .1$



Regular Scalar  $\mathfrak{R}^2$  for  $s = \frac{4}{3}$ :

$$\mathfrak{R}^2_{\text{perfect fluid}} = \frac{8(5w^{16} + 20w^{12} + 174w^8 + 20w^4 + 5)}{(1 + w^4)^4}$$

$$w = v/\Delta^{\frac{1}{4}}(\frac{4}{3}) \equiv \sqrt[4]{3}v.$$

# Moving Black Hole Dual of a Perfect Relativistic fluid

$$v = \frac{z}{\tau^{1/3}}$$

- Asymptotic (Fefferman-Graham) metric

$$ds^2 = \frac{1}{z^2} \left[ -\frac{\left(1 - \frac{e_0}{3} \frac{z^4}{\tau^{4/3}}\right)^2}{1 + \frac{e_0}{3} \frac{z^4}{\tau^{4/3}}} d\tau^2 + \left(1 + \frac{e_0}{3} \frac{z^4}{\tau^{4/3}}\right) (\tau^2 dy^2 + dx_\perp^2) \right] + \frac{dz^2}{z^2}$$

- Interpretation: Black Hole moving off in the 5th dimension (in FF-G coordinates)

$$\text{Horizon : } z_0 = \left(\frac{3}{e_0}\right)^{\frac{1}{4}} \cdot \tau^{\frac{1}{3}} .$$

$$\text{Temperature : } T(\tau) \sim \frac{1}{z_0} \sim \tau^{-\frac{1}{3}}$$

$$\text{Entropy : } S(\tau) \sim \text{Area} \sim \tau \cdot \frac{1}{z_0^3} \sim const$$

- In  $1 \rightarrow 1$  correspondence with Bjorken flows

# In-flow Viscosity and Relaxation time

R.Janik, R.Janik and M.Heller

- Shear Viscosity equation (first order)

$$\partial_\tau \epsilon = -\frac{4}{3} \frac{\epsilon}{\tau} + \frac{\eta}{\tau^2}$$

- Asymptotic Expansion of the Black Hole Solution

$$a(\tau, z), b(\tau, z), c(\tau, z) \Rightarrow \sum_n \lambda_n^{a,b,c}(v) \tau^{-2n/3}$$

$$\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} \Rightarrow \sum_n \mathfrak{R}^2_n \tau^{-2n/3}$$

- Results

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Same as static (needs  $n \rightarrow 2$  !)

$$\tau_{Rel} = (1 - \log 2)/6\pi T :$$

Relaxation Time

(2nd order formalism, needs  $n \rightarrow 3$  !)

$$\langle \text{Tr}F^2 \rangle < 0 :$$

Einstein-Dilaton System; Magnetic > Electric

# QGP/BH Duality beyond Perfect fluid

Dynamic Case

- Going beyond perfect fluid

*In-flow Viscosity, Relaxation time, Transport Coeff., etc...*

Janik, Heller, Bak, Benincasa, Buchel, Nakamura, Sin,.....  
Kinoshita, Mukoyama, Nakamura, Oda, Natsuume, Okamura,...

$$\partial_\tau \epsilon = -\frac{4}{3} \frac{\epsilon}{\tau} + \frac{\eta}{\tau^2} + \dots \Rightarrow \frac{\eta}{s} = \frac{1}{4\pi}$$

- Going beyond boost-invariance

*Fluid/Gravity Duality*

Bhattacharyya, Hubeny, Minwalla, Ranganami, Loganayagam,...

$$T_{rescaled}^{\mu\nu} = \underbrace{(\pi T)^4 (\eta^{\mu\nu} + 4u^\mu u^\nu)}_{\text{perfect fluid}} - \underbrace{2(\pi T)^3 \mu^\nu}_{\text{viscosity}} + \\ + \underbrace{(\pi T^2) \left( \log 2T_{2a}^{\mu\nu} + 2T_{2b}^{\mu\nu} + (2 - \log 2) \left( \frac{1}{3} T_{2c}^{\mu\nu} + T_{2d}^{\mu\nu} + T_{2e}^{\mu\nu} \right) \right)}_{\text{second order hydrodynamics}}$$

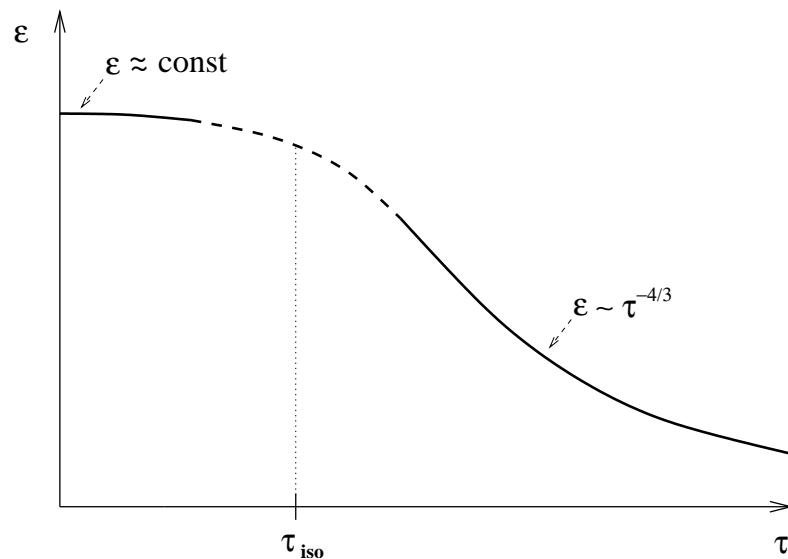
- Going Beyond hydrodynamics? Far Before equilibrium? Early times?

# Lecture III: QGP/BH Duality: Early-time Flow

## First Approaches (1)

- Scaling at early vs. late proper-time

Kovchegov, Taliotis (2007)



- Evaluation of The Isotropization/Thermalization time

$$\text{Matching} : z_h^{\text{late}}(\tau) = \left( \frac{3}{e_0} \right)^{\frac{1}{4}} \equiv z_h^{\text{early}}(\tau) = \tau$$

$$\text{Isotropization} : \tau_{\text{iso}} = \left( \frac{3N_c^2}{2\pi^2 e_0} \right)^{3/8}$$

$$\text{Typical Scale} : \epsilon(\tau) = e_0 \tau^{4/3} |_{\tau=.6} \sim 15 \text{ GeV fermi}^{-3}$$

$$\Rightarrow \boxed{\tau_{\text{iso}} \sim .3 \text{ fermi}}$$

# Thermalization “response-time” of a perfect fluid

## First Approaches (2)

- Quasi-normal scalar modes of a static Black Hole

G.Horowitz,V.Hubeny

$$\boxed{\Delta\phi \equiv \frac{1}{\sqrt{-g}} \partial_n (\sqrt{-g} g^{ij} \partial_j \phi) = 0}$$

(here in Fefferman-Graham coordinates)

$$-\frac{1}{z^3} \frac{\left(1 - \frac{z^4}{z_0^4}\right)^2}{\left(1 + \frac{z^4}{z_0^4}\right)} \partial_t^2 \phi(t, z) + \partial_z \left( \frac{1}{z^3} \left(1 - \frac{z^8}{z_0^8}\right) \partial_z \phi(t, z) \right) = 0 .$$

- Separation of variables  $\phi(t, z) = e^{i\omega t} \phi(z)$

$$\phi'' + \frac{1 - \tilde{z}^2}{\tilde{z}(1 - \tilde{z})(2 - \tilde{z})} \phi' + \left(\frac{\omega}{\pi T}\right)^2 \frac{1}{4\tilde{z}(1 - \tilde{z})(2 - \tilde{z})} \phi = 0$$

- Dominant Decay Time

$$\frac{\omega_c}{\pi T} \sim 3.1194 - 2.74667 i$$

# Thermalization response-time of an expanding fluid

R.Janik,R.P.

- Quasinormal scalar modes for the boost-invariant Black Hole Geometry

$$\Delta\phi \equiv \frac{1}{\sqrt{-g}} \partial_n (\sqrt{-g} g^{ij} \partial_j \phi) = 0$$

$$-\frac{1}{v^3} \frac{(1+v^4)^2}{1-v^4} \partial_\tau^2 \phi(\tau, v) + \tau^{-\frac{2}{3}} \partial_v \left( \frac{1}{v^3} (1-v^8) \partial_v \phi(\tau, v) \right) = 0$$

- Separation of variables  $\phi(\tau, v) = f(\tau)\phi(v)$

$$\partial_\tau^2 f(\tau) = -\omega^2 \tau^{-\frac{2}{3}} f(\tau) \Rightarrow f(\tau) = \sqrt{\tau} J_{\pm\frac{3}{4}} \left( \frac{3}{2} \omega \tau^{\frac{2}{3}} \right) \sim \tau^{\frac{1}{6}} e^{\frac{3}{2} i \omega \tau^{\frac{2}{3}}}$$

$$\partial_v \left( \frac{1}{v^3} (1-v^8) \partial_v \phi(v) \right) + \omega^2 \frac{1}{v^3} \frac{(1+v^4)^2}{1-v^4} \phi(v) = 0$$

- Short Decay Proper-Time

$$\frac{\omega_c}{\pi T} \sim 3.1194 - 2.74667 i \Rightarrow \tau \sim \frac{1}{8.3 T}$$

- Evolution at early vs. late proper-time

Gubser's guess, (2009)

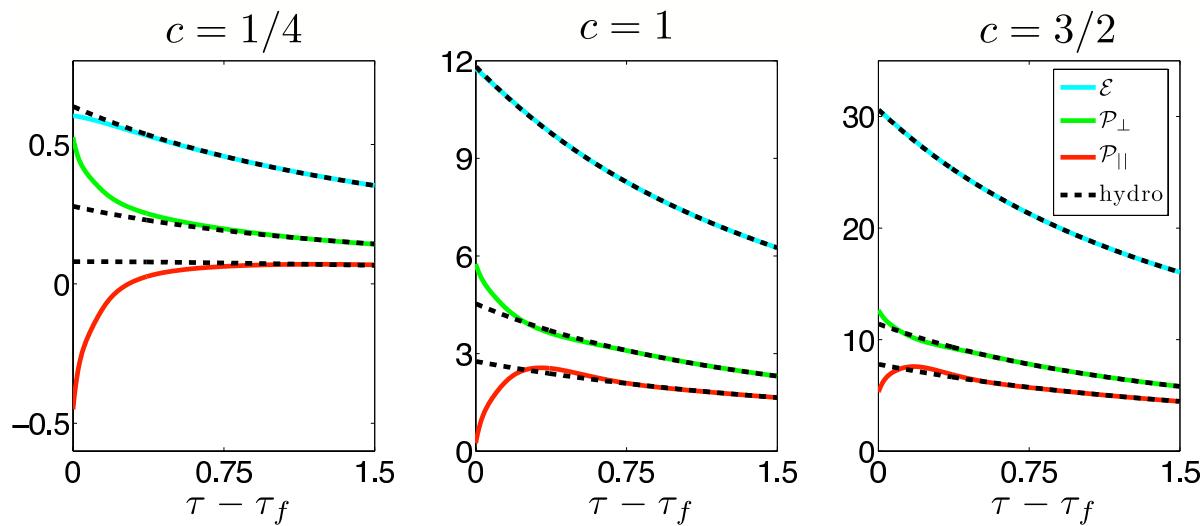
$$T \sim T_c \approx 150-200 \text{ MeV} \otimes 3-4 \text{ Iterations} \Rightarrow \tau_{therm} \approx .5-1 \text{ fermi}$$

# Far-from-equilibrium forcing Dynamics

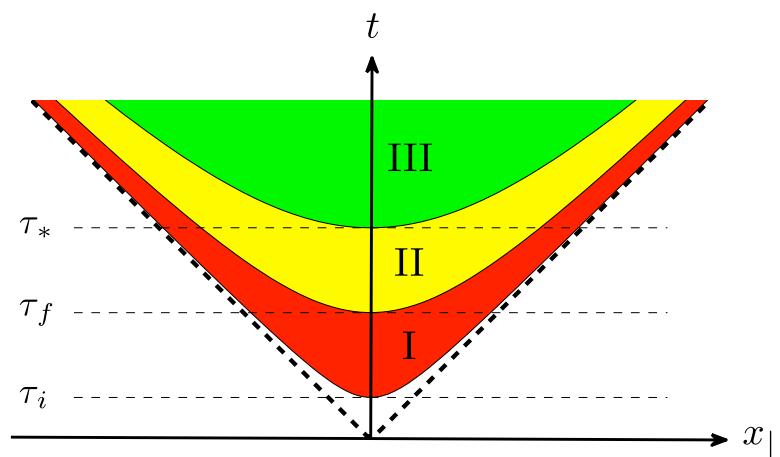
P.M.Chesler, L.G.Yaffe, 2009

- 4d Time-dependent Shear

$$ds^2 = -d\tau^2 + \tau^2 e^{-2c\gamma(\tau)} dy^2 + e^{c\gamma(\tau)} dx_\perp^2$$

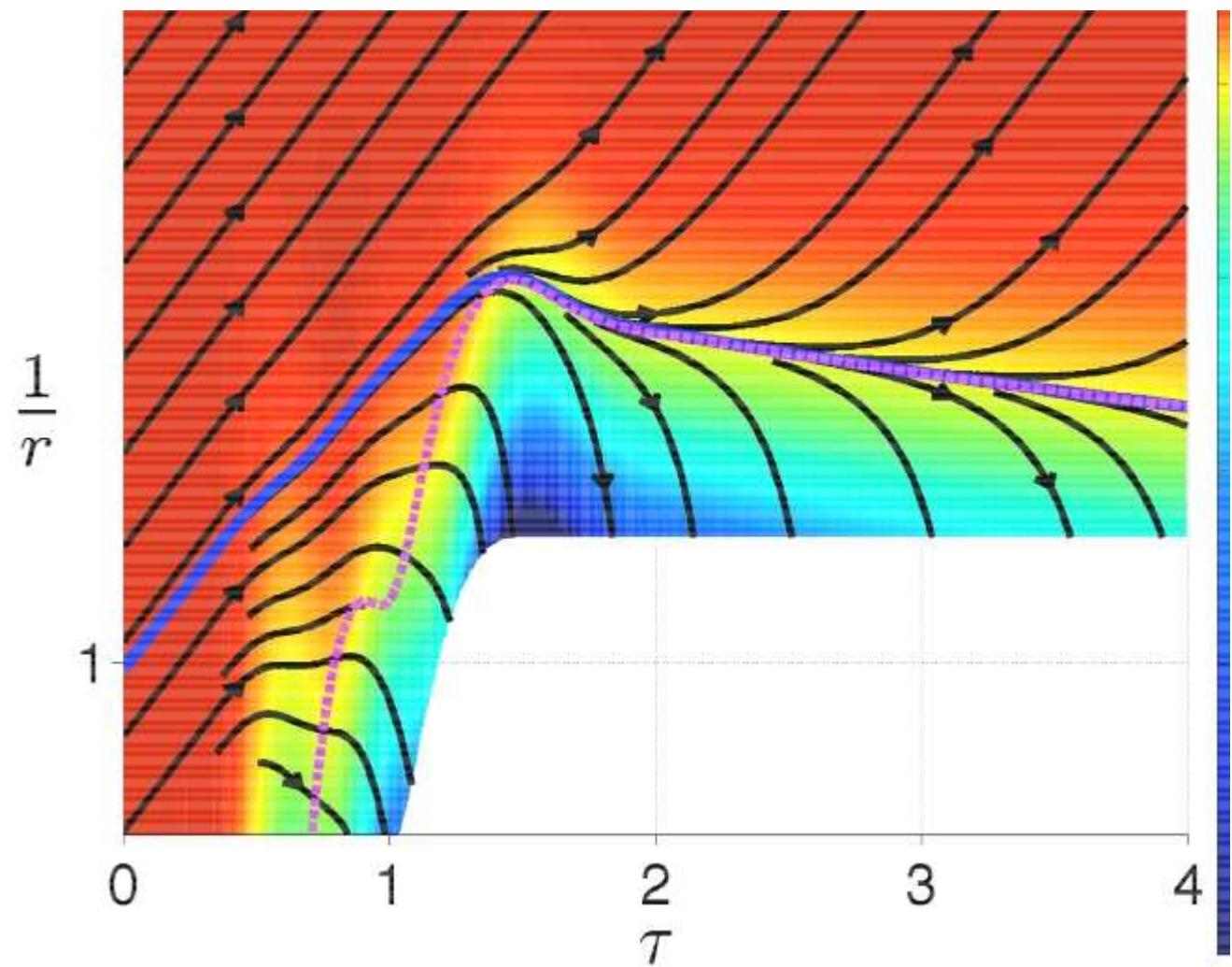


- 5d Black hole Formation



$I$  : 4d Deformation    $II$  : Anisotropic Relaxation    $III$  : Hydro Regime

## Black Hole Formation



# QGP/BH Duality: Early-time Flow

G.Beuf, M.Heller, R.Janik, R.P., 2009

- General Boost-Invariant Fefferman-Graham metric:

$$ds^2 = \frac{-e^{a(\tau,z)} d\tau^2 + \tau^2 e^{b(\tau,z)} dy^2 + e^{c(\tau,z)} dx_\perp^2}{z^2} + \frac{dz^2}{z^2}$$

- Einstein Equation:

$$R_{AB} + 4G_{AB} = 0$$

- To be solved:  $(\dot{a} = \partial_\tau a; a' = \partial_z a; \dots)$

$$(\tau\tau) : \ddot{b} + 2\ddot{c} - \frac{\dot{a}}{2}(\dot{b} + 2\dot{c}) + \frac{1}{2}(\dot{b}^2 + 2\dot{c}^2) - \frac{1}{\tau}(\dot{a} - 2\dot{b}) = e^a \left\{ a'' - \frac{3a'}{z} + \left( \frac{a'}{2} - \frac{1}{z} \right) (a' + b' + 2c') \right\}$$

$$(yy) : \ddot{b} - \dot{a}\dot{b} + \frac{1}{\tau}(\dot{b} - 2\dot{a}) + \frac{1}{2}(\dot{a} + \dot{b} + 2\dot{c}) \left( \dot{b} + \frac{2}{\tau} \right) = e^a \left\{ b'' - \frac{3b'}{z} + \left( \frac{b'}{2} - \frac{1}{z} \right) (a' + b' + 2c') \right\}$$

$$(\perp\perp) : \ddot{c} - \dot{a}\dot{c} + \frac{\dot{c}}{2} \left( \dot{a} + \dot{b} + 2\dot{c} + \frac{2}{\tau} \right) = e^a \left\{ c'' - \frac{3c'}{z} + \left( \frac{c'}{2} - \frac{1}{z} \right) (a' + b' + 2c') \right\}$$

$$(\tau z) : 2\dot{b}' + 4\dot{c}' + b' \left( \dot{b} + \frac{2}{\tau} \right) + 2\dot{c}\dot{c}' - a' \left( \dot{b} - 2\dot{c} + \frac{2}{\tau} \right) = 0$$

$$(zz) : a'' + b'' + 2c''' - \frac{1}{z}(a' + b' + 2c') + \frac{1}{2}(a'^2 + b'^2 + 2c'^2) = 0$$

## Early-Time; General Features

- Dependence on Initial Conditions

$$a(\tau, z) = \dots + z^8 \left\{ -1/16 \tau^{-2s} s^2 - 1/6 \tau^{-2s} + 1/6 \tau^{-2s} s \right\} + \frac{z^4}{\tau^s} \left\{ \frac{1}{96} \frac{z^4}{\tau^4} s^2 - \frac{1}{384} \frac{z^4}{\tau^4} s^4 \right\} + \dots$$

scaling part  $\{\dots\}$  not dominant when  $s=0$

- The metric is singular at all times (including  $\tau = 0$  !):

$$\text{Set : } u(z^2) = \frac{1}{4z} a'_0(z) \quad v(z^2) = \frac{1}{4z} b'_0(z) \quad w(z^2) = \frac{1}{4z} c'_0(z)$$

$$0 = [u + v + 2w]_0^\infty \equiv \int_0^\infty (u' + v' + 2w') dz^2 \neq -2 \int_0^\infty (u^2 + v^2 + 2w^2) dz^2$$

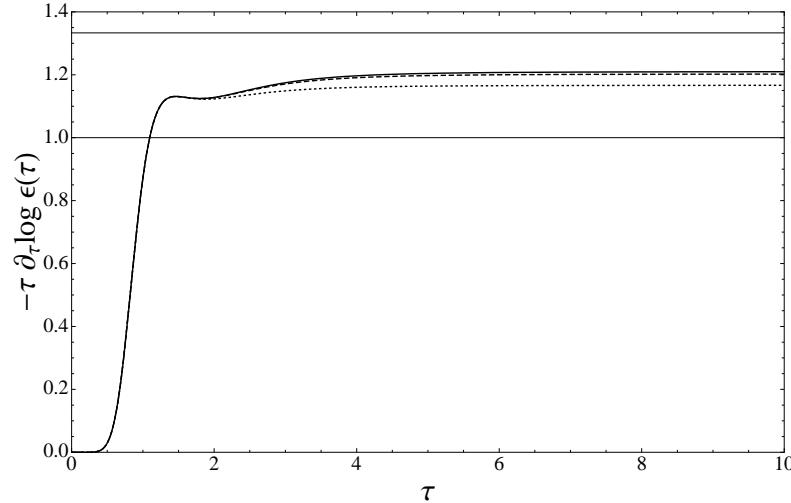
- “MicroCosmic Censorship” :

$$ds^2(z \sim z_{sing}) \sim \frac{1}{z^2} \left( 1 - \frac{z}{z_{sing}} \right)^2 d\tau^2 + \dots + \frac{1}{z^2} dz^2$$

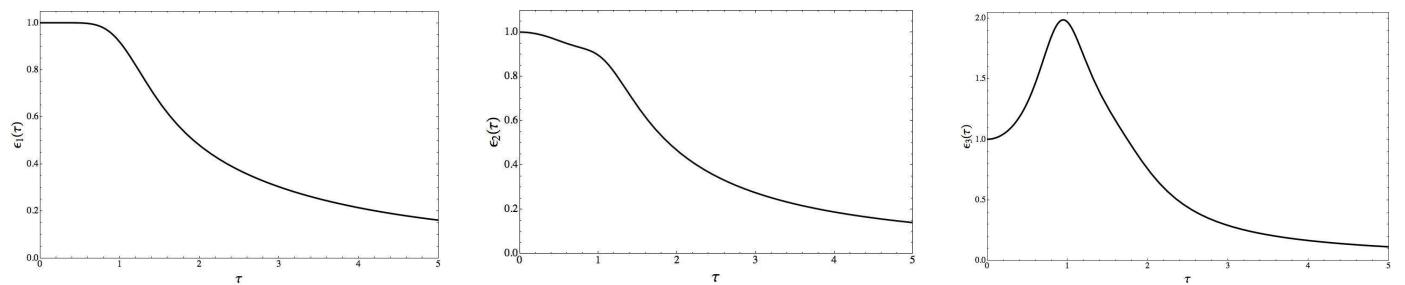
- To be satisfied: [Initial Conditions + Constraints]

# Investigations on Thermalization

- “Family Index”:  $s = -\tau \partial_\tau \epsilon(\tau)$



- Dependence on Initial Conditions



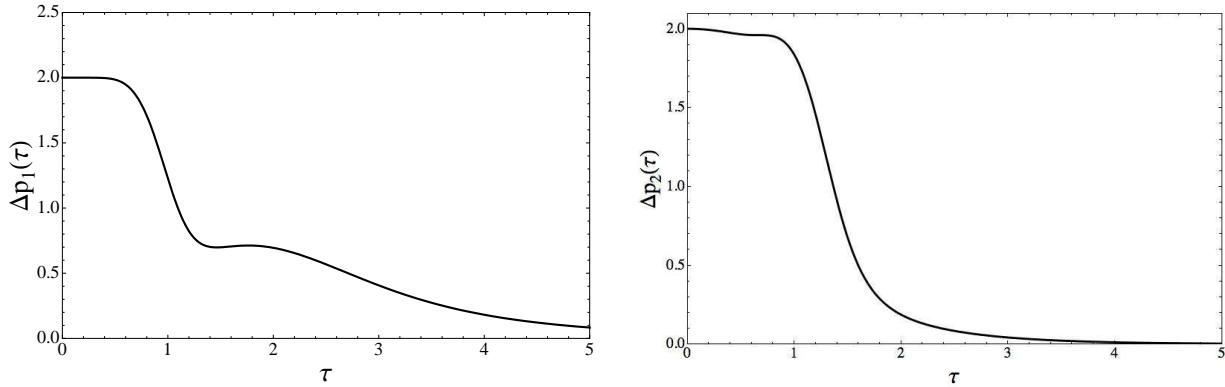
$$[v + w - >]A) : \tanh(z^2) - \tan(z^2) \quad B) : \tanh(z^2 + z^8/6) - \tan(z^2) \quad C) : 2/3z^6(1+z^2/2)/(z^2-1)$$

$$T_{\mu\nu} t^\mu t^\nu \geq 0 \Rightarrow$$

*C): Temporary violation of Positivity:*

$$\frac{4\epsilon(\tau)}{\tau} \leq \epsilon'(\tau) \leq 0$$

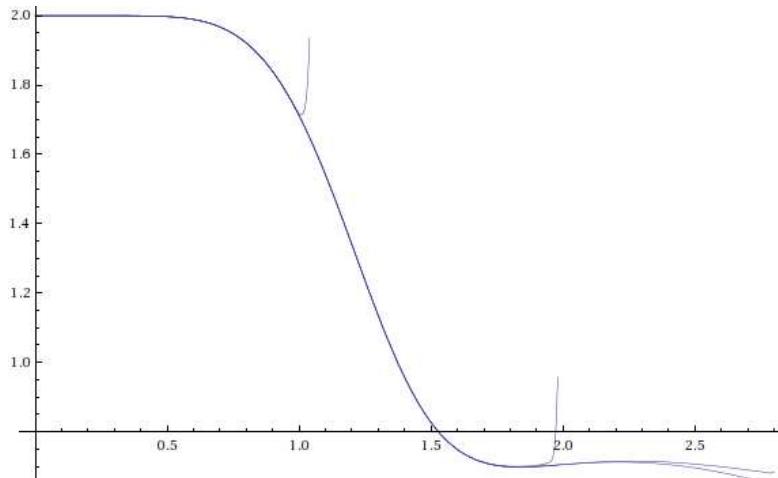
# Isotropization of Pressure Density



$$[v + w - >]A) : \tanh(z^2) - \tan(z^2) \quad B) : \tanh(z^2 + z^8/6) - \tan(z^2)$$

$$\boxed{\Delta p(\tau) = 1 - \frac{p_{\parallel}(\tau)}{p_{\perp}(\tau)}}$$

- “Fast” Road towards Isotropization but:
- Isotropization may stay “some time” incomplete

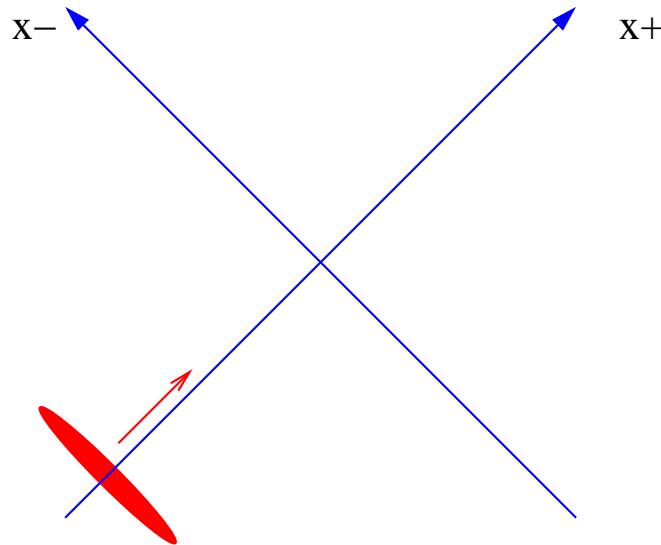


- $\Rightarrow$  Interesting geometries of Black Hole formation

# Shock Waves

- Describing the Fast Nucleus in AdS/CFT

Janik, R.P.(2005)



- The Dual Shock Wave

$$ds^2 = \frac{-2dx^+dx^- + \mu_1 z^4 F(x^-) \{ = \delta(x^-) \} dx^{-2} + d\mathbf{x}_\perp^2 + dz^2}{z^2}$$

Extension:  $F(x^-) \rightarrow F(x^-, x_\perp, z)$  G.Beuf (2009)

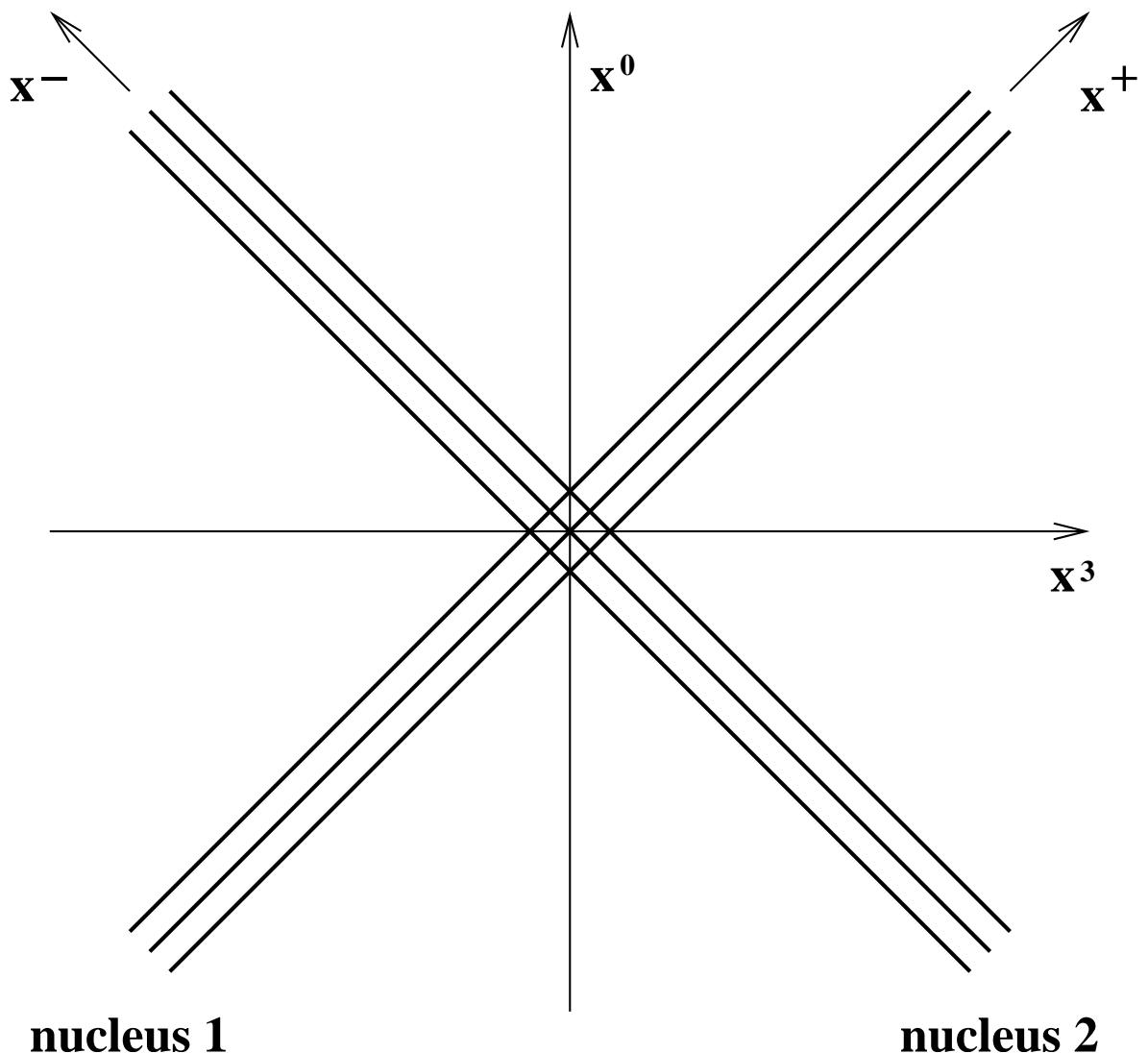
- Dipole-Shock Wave Scattering

cf. Albacete,Kovchegov,Taliotis; Iancu,Mueller... (2008)

# Shock-Wave Collisions

An Open problem  
(2008 → ⋯)

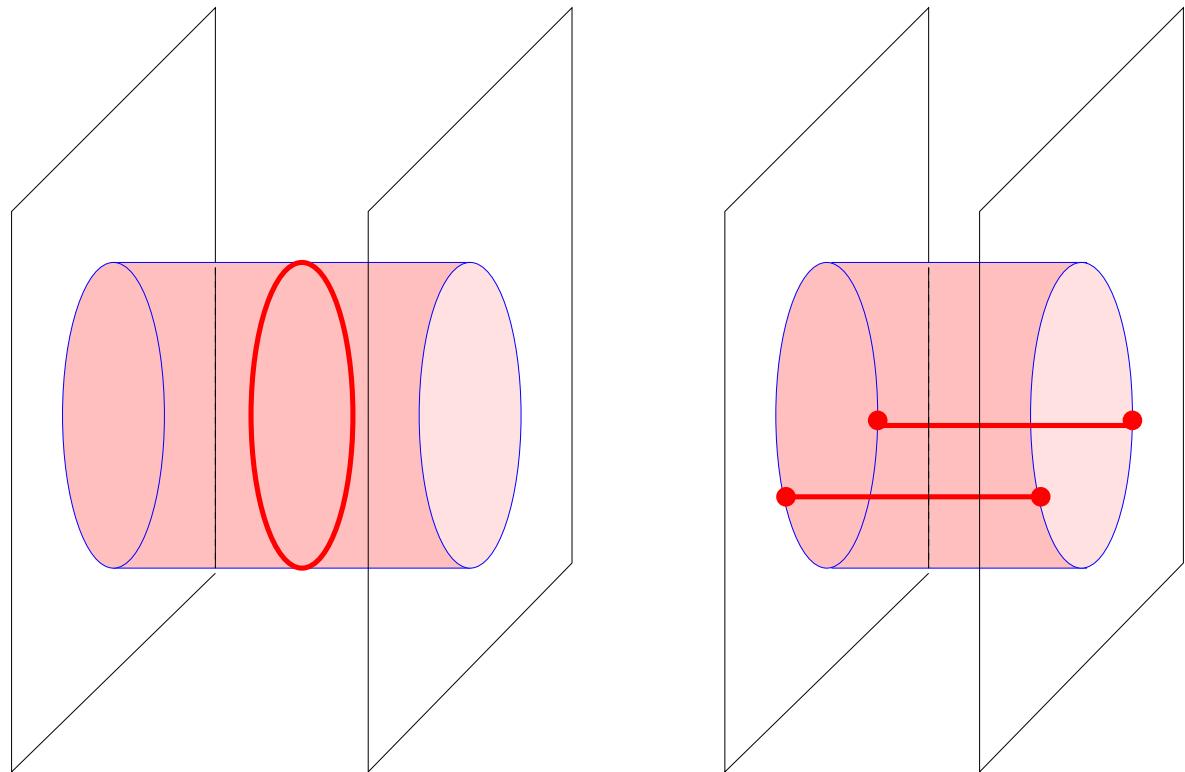
Grumiller, Romatschke ; Albacete, Kovchegov, Taliotis, ...



# (1/2) Lecture IV: AdS/CFT and the cosmic flow

Gauge-Gravity Correspondence: the other facet

Gauge  $\Rightarrow$  Gravity



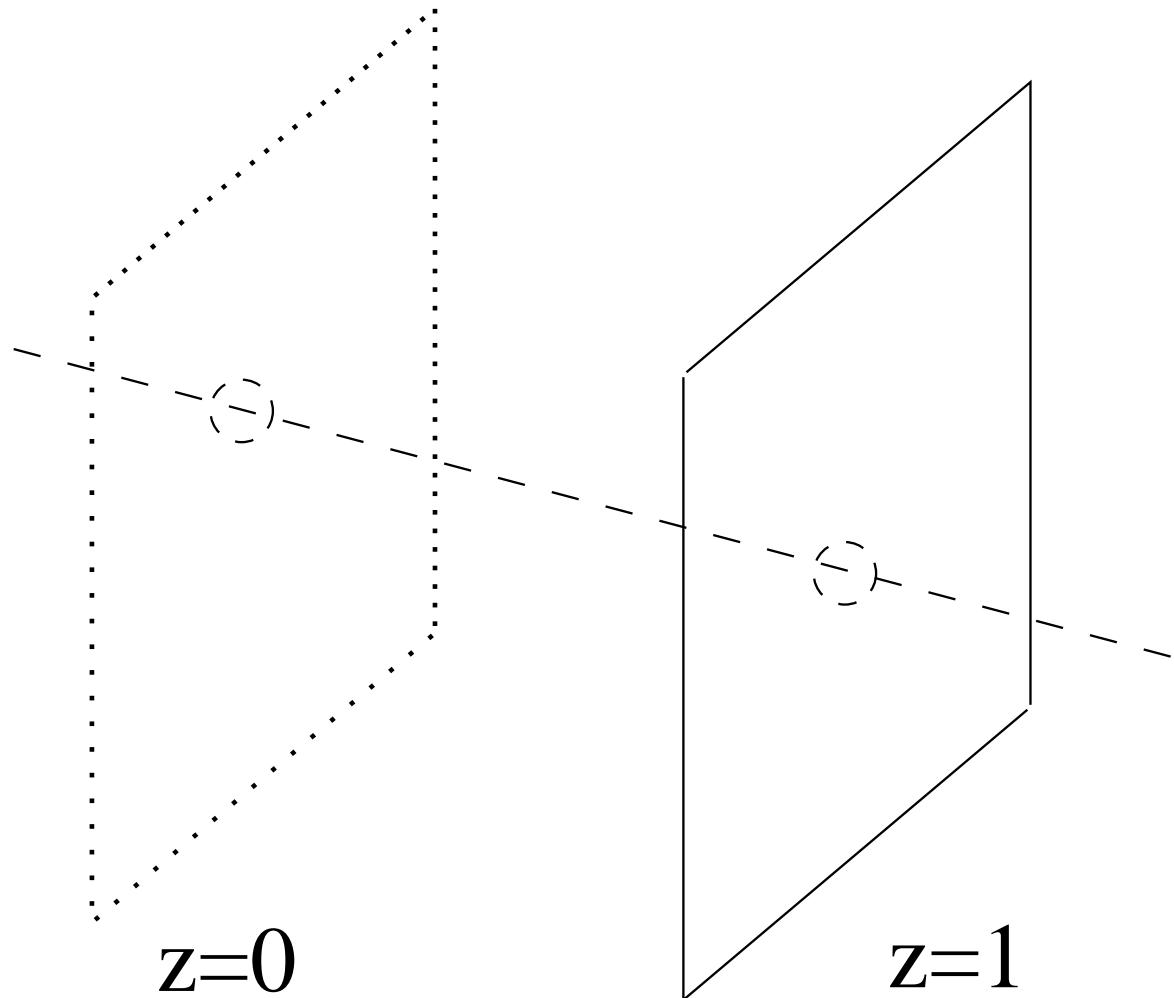
*Closed String*  $\Leftrightarrow$  *1-loop Open String*

*Gravity*  $\Leftrightarrow$  *Gauge*

*D-Brane “Universe”*  $\Rightarrow$  *Open String Ending*

*Small/Large Distance*  $\Rightarrow$  *Gauge/Gravity Correspondence*

# The Brane/Bulk/Brane Geometry



$z = 1 \Rightarrow \text{Cosmology Brane}$

$z = 0 \Rightarrow \text{Holographic Brane (AdS Boundary)}$

$z > 1 \Rightarrow \text{Bulk } AdS_5 \text{ Gravity}$

$0 < z < 1 \Rightarrow z \rightarrow 1/z \text{ Mirror } AdS_5$

$\text{Brane} \leftrightarrows \text{Brane} \Rightarrow \text{Gauge/Cosmology Duality}$

# Holography at work

Brane → Bulk: Holographic Renormalization

K.Skenderis (2002)

- Bulk metric

$$ds^2 = \frac{g_{\mu\nu}(z) dx^\mu dx^\nu + dz^2}{z^2}$$

(in Fefferman-Graham Coordinates)

- $5d \Rightarrow 4d$  metric:

$$g_{\mu\nu}(z) = g_{\mu\nu}^{(0)} (\neq \eta_{\mu\nu}) + z^2 g_{\mu\nu}^{(2)} (\neq 0) + z^4 g_{\mu\nu}^{(4)} (\neq \langle T_{\mu\nu} \rangle) + z^6 \dots +$$

+z<sup>6</sup> ... +: from Einstein Eqs.

- Energy-Momentum Tensor:

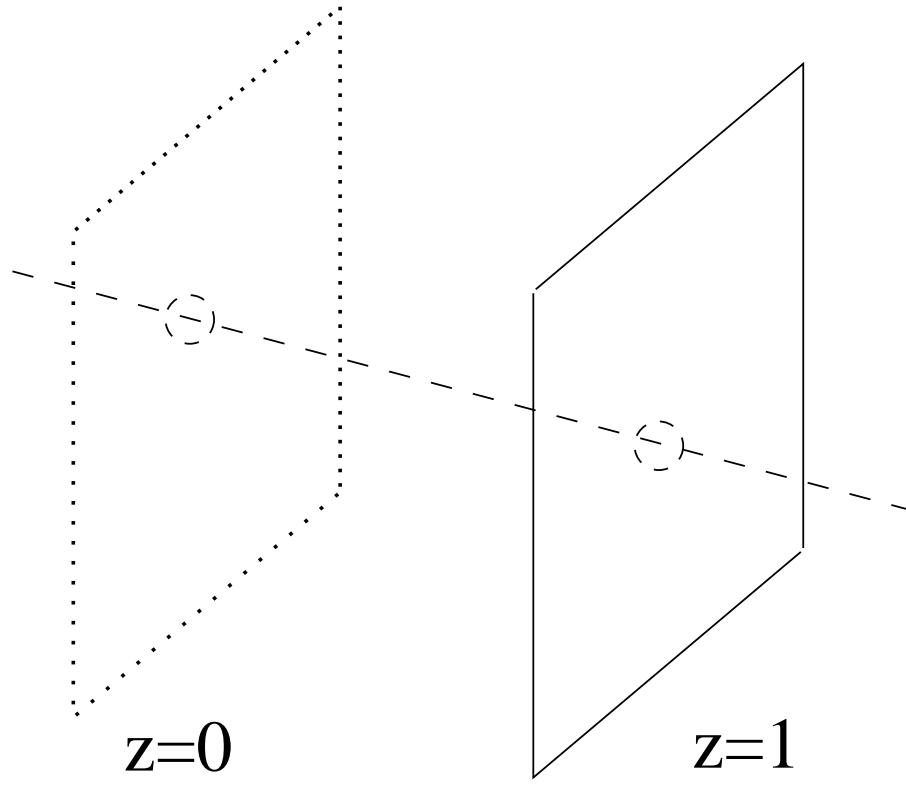
$$\langle T_{\mu\nu} \rangle = \frac{2l^3}{\kappa_5^2} \left\{ g_{\mu\nu}^{(4)} - \frac{1}{8} g_{\mu\nu}^{(0)} \left[ (\text{Tr}(g^{(2)}))^2 - (\text{Tr}g^{(2)})^2 \right] - \frac{1}{2} g_{\mu\rho}^{(2)} g^{(0)\rho\sigma} g_{\sigma\nu}^{(2)} + \frac{1}{4} (\text{Tr}g^{(2)}) g_{\mu\nu}^{(2)} \right\}$$

Note: Curved 4d metric, non Minkowskian ( $\rightarrow$  FRW)

# The Cosmology Brane

- Brane-World Cosmology

Binetruy, Deffayet, Ellwanger, Langlois (2000)



$$\langle T \rangle_B = \text{diag}(-\rho_B, \rho_B, \rho_B, \rho_B, \rho_B) \Rightarrow \langle T \rangle_b = \text{diag}(-\rho_b, p_b, p_b, p_b, p_b)$$

(tuning  $\Lambda$  with Israel Conditions)

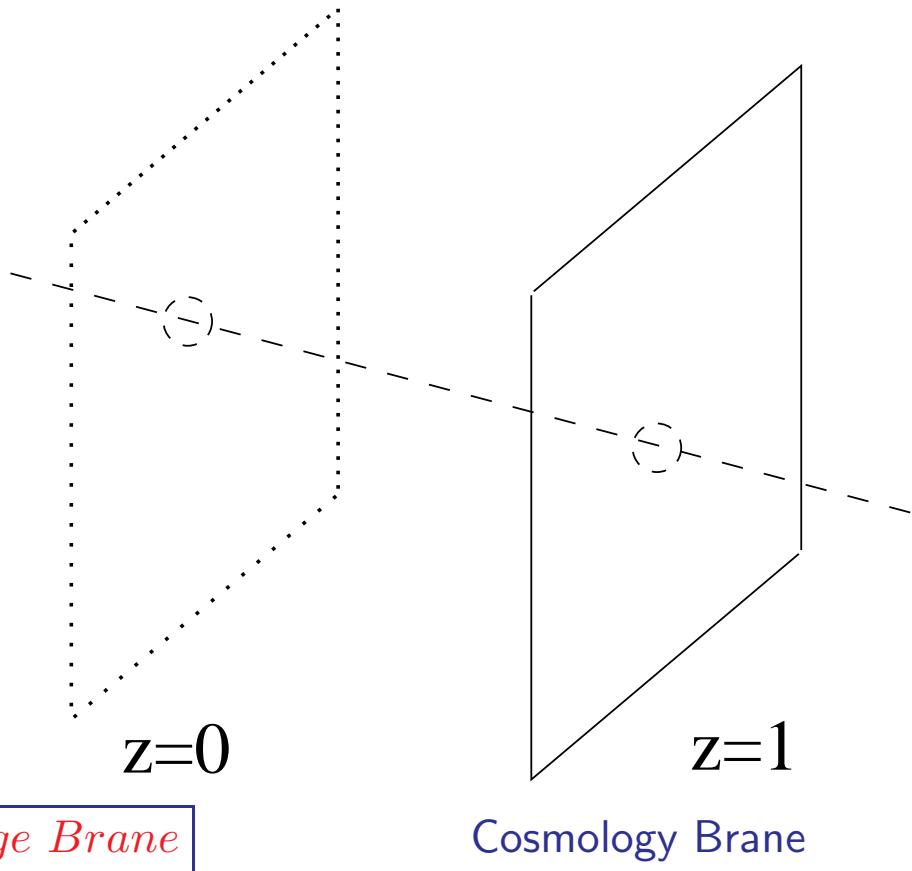
- Induced Friedmann equation

$$H^2 l^2 = -1 + \frac{\rho_b^2}{\rho_\Lambda^2} + \frac{C}{a_0^4}$$

# The Gauge Brane

- Isotropically Expanding Plasma in AdS/CFT

Kajantie, Tahkokallio (2007)



Gauge Brane

Cosmology Brane

$$ds_5^2 = \frac{l^2}{z^2} \left[ dz^2 - \frac{dt_K^2}{l^2} \frac{h^2 r^2}{b(t_K, z)} (1 + A_2 z^2 + A_4 z^4)^2 + \frac{dx^2}{l^2} b(t_K, z) \right]$$

$b(t_K, z), A_2, A_4$ , functions of  $r, h$ , and  $t_K$  derivatives

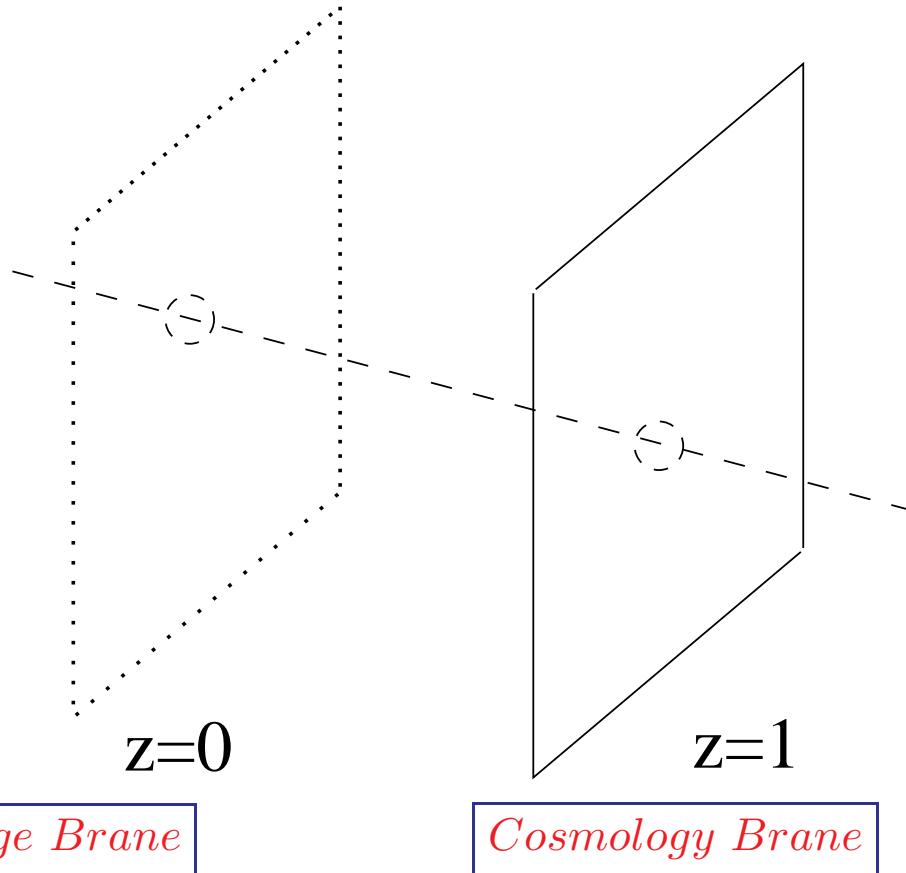
- Gauge/Gravity duality

$$\frac{l^3}{\kappa_5^2} = \frac{N_c^2}{4\pi^2}$$

# Brane-to-Brane Duality

- Isotropically Expanding Plasma in AdS/CFT

Brax, R.P. (2010)



$$ds_5^2 = \frac{l^2}{z^2} \left[ dz^2 - \frac{dt_B^2}{l^2} \left( \frac{H^2 l^2/2 + \dot{H} l^2}{2} - z^2 \right)^2 + \frac{dx^2}{l^2} a_0^2(t_B) \left( \frac{H^2 l^2}{4} - z^2 \right)^2 \right]$$

- Brane-to-Brane Space-Time Relations

$$r(t_K) = \frac{H^2 l^2}{4} a_0(t_B) \quad h \frac{dK}{dt_B} = \epsilon_K \frac{dr}{da_0}$$

# Gauge/Cosmology Duality

- $\mathcal{N} = 4$  SYM Energy-Momentum Tensor

$$T_b^{\mu\nu} = (\epsilon_b + p_b) u^\mu u^\nu - p_b \eta^{\mu\nu}$$

- Duality Relations

$$\frac{r^2}{a_0^2} = \frac{1}{4} \left\{ \frac{\rho^2}{\rho_\Lambda^2} + \frac{\mathcal{C}}{a_0^4} \right\} : \quad \rho_b \equiv \rho_\Lambda + \rho$$

$$\rho_K = \frac{3N_c^2}{8\pi^2} \frac{a_0^4}{r^4} \left( \frac{\mathcal{C}}{a_0^4} + \frac{H^4 l^4}{4} \right) : \quad \text{Energy density}$$

$$\rho_K - 3p_K = \frac{\ddot{a}_0}{a_0} \frac{3N_c^2}{8\pi^2} \left( \frac{a_0}{r} \right)^3 \epsilon_K \frac{da_0}{dr} : \quad \text{Trace Anomaly}$$

- Covariant Acceleration/Anomaly Relation

$$\sqrt{-g_B} dt_B H^2 \frac{\ddot{a}_0}{a_0} = \sqrt{-g_K} dt_K \frac{8\pi^2}{3N_c^2} (\rho_K - 3p_K)$$

# Plasma/Cosmology Duality

- Equations of State: Holographic brane

$$w_{eff} = \frac{p_H}{\rho_H} = -\frac{w}{2+3w}$$

- Duality Relations

Matter Cosmology  $\omega = 0 \Leftrightarrow w_{eff} = 0$

Dark energy  $\omega = -1 \Leftrightarrow w_{eff} = -1$

No acceleration  $\omega = -1/3 \Leftrightarrow w_{eff} = 1/3 = \text{Perfect Fluid}$

- Holographic expansion/contraction

$$a_0 \sim t_B^{\frac{2}{3(1+w)}} \Leftrightarrow a_H(t) \sim t_K^{\frac{2}{3(1+w_{eff})}} = t_K^{-\frac{2(2+3w)}{3(1+w)}}$$

Matter Cosmology  $\omega > -2/3 \Leftrightarrow \text{Plasma Contraction}$

Dark energy  $\omega < -2/3 \Leftrightarrow \text{Plasma Expansion}$

## “Table of Signs”

Positivity Constraints:  $\frac{dt_K}{dt_B}, \rho_K - 3p_K \geq 0$

$\omega$	-1		$-\frac{2}{3}$		$-\frac{1}{3}$		0		$\frac{1}{3}$
$\rho$ vs. $\mathcal{C}$		$\rho$		$\rho$		$\mathcal{C}$		$\mathcal{C}$	
sign( $h$ )		+		+		-		-	
$\epsilon_K$		+		-		+		+	
sign( $\ddot{a}_0$ )		+		+		-		-	
sign( $\frac{dr}{da_0}$ )		+		-		-		-	

$1^{rst}$  line: relative dominance of the matter density over the dark radiation in the holographic map  $r(a_0)$ ;

$2^{nd}$  line: sign of the lapse function  $h$ ;

$5^{th}$  line: value of  $\epsilon_K$  .

$3^{rd}$  line: sign of the cosmic acceleration;

$4^{th}$  line: expansion (+) vs. contraction (-) of the dual plasma phase;

## An intriguing Relation:

$$\mathcal{A} \equiv \lim_{N_c \rightarrow \infty} \frac{8\pi^2}{3} \frac{\rho_H - 3p_H}{N_c^2} = \mathcal{E}^4$$

Anomaly by d.o.f.

$$dN_B = H dt_B = d \log a_0$$

Number of e-foldings

$$\Delta\tau\Delta\mathcal{E} \sim \Delta N_B = O(1)$$

Quantum/Classical Identity