The moduli space of partially broken $N = 2$ supergravities

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in collaboration with

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Vicente Cortés, Paul Smyth & Hagen Triendl in preparation

"Advances in string theory, wall crossing, and quaternion-Kähler geometry"

Paris, September 2010

History of spontaneous $N = 2 \rightarrow N = 1$ breaking

- '84: [Cecotti,Girardello,Porrati] showed:

  "Two into one won't go"

  Minkowski vacua of $4d, N = 2$ supergravities either have full $N = 2$ or $N = 0$ supersymmetry.

- '95 way out: include magnetic FI-term/magnetic charges

  [Antoniadis,Partouche,Taylor; Ferrara,Girardello,Porrati]

  few explicit examples constructed

- recently systematic analysis [Smyth,Triendl,JL]

  using embedding tensor formalism of [deWit,Samtleben,Trigiante]

Outline of the talk

1. brief review of $N = 2$ supergravity
   and the embedding tensor formalism

2. formulate the conditions of partial supersymmetry breaking
   in the embedding tensor formalism

3. discuss solutions

4. derive $N = 1$ effective action and discuss its moduli space

5. apply to type II string theory

$N = 2$ supergravity

➪ spectrum

gravity multiplet: $\left[\mathbf{g}_{\mu\nu},\ \mathbf{\Psi}_{\mu\mathcal{A}},\ \mathbf{A}^{\mathbf{0}}_{\mu}\right]$, $\quad \mathcal{A}{=}1{,}2$

vector multiplets: $\left[\mathbf{A}^{\mathbf{i}}_{\mu},\ \lambda^{\mathbf{i}\mathcal{A}},\ \mathbf{t}^{\mathbf{i}}\right]$, $\quad i{=}1,\ldots,n_{\rm v}$

hypermultiplets: $\left[\zeta_{\alpha},\ \mathbf{q}^{\mathbf{u}}\right]$, $\quad \alpha{=}1,\ldots,2n_{\rm h}$, $\quad u{=}1,\ldots 4n_{\rm h}$

➪ scalar field space

$$\mathbf{M} = \mathbf{M}_{\mathbf{v}}^{\mathbf{SK}}(\mathbf{t}) \times \mathbf{M}_{\mathbf{h}}^{\mathbf{QK}}(\mathbf{q})$$

Special Kähler manifolds

$\mathbf{M_v^{SK}(t)}$: special Kähler manifold [de Wit,Van Proeyen]

- Kähler metric determined by holomorphic prepotential $\mathbf{F}$

  $$\mathbf{K}_{\rm v} = -\ln \mathbf{i}\left[\mathbf{\bar{X}^I(\bar{t})F_I(t) - X^I(t)\bar{F}_I(\bar{t})}\right] , \quad \mathbf{F_I} \equiv \partial_\mathbf{I}\mathbf{F} , \qquad \mathbf{I,=0,\ldots,n_{\rm v}}$$

  special coordinates: $\mathbf{X^I = (1, t^i)}$

- $\mathbf{Sp(2n_{\rm v}+2)}$ electric-magnetic duality rotations act on symplectic vectors $(\mathbf{A}^\mathbf{I}_\mu, \mathbf{\tilde{A}}_{\mu\mathbf{I}})$ and $\mathbf{V^\Lambda = (X^I, F_I)}$, $\mathbf{\Lambda=0,\ldots,2n_{\rm v}}$

Quaternionic Kähler manifold

$\mathbf{M_h^{QK}(q)}$: quaternionic Kähler manifold [Bagger, Witten,...]

- triplet of $SU(2)$-covariantly closed almost complex structures $\mathbf{J^x(q)}$ exists

$$\nabla \mathbf{J^x} = \mathbf{0} , \qquad \mathbf{J^x J^y} = -\delta^{\mathbf{xy}} \mathbf{Id} + \epsilon^{\mathbf{xyz}} \mathbf{J^z} , \qquad \mathbf{x=1,2,3}$$

associated triplet of Kähler-forms

$$\mathbf{K^x} = \mathbf{d}\omega^{\mathbf{x}} + \frac{\mathbf{1}}{\mathbf{2}} \epsilon^{\mathbf{xyz}} \omega^{\mathbf{y}} \wedge \omega^{\mathbf{z}} , \qquad \omega : SU(2) - \text{connection}$$

- Killing vectors $\mathbf{k}_\lambda$ determined in terms of triplet of Killing prepotentials $\mathbf{P^x}$ (moment maps)

$$\mathbf{k}_\lambda \cdot \mathbf{J^x} = \nabla \mathbf{P}^{\mathbf{x}}_\lambda$$

Embedding Tensor formalism

[de Wit,Samtleben,Trigiante]

introduce simultaneously into the action:

- electric gauge field: $\mathbf{A}_\mu^{\mathbf{I}}$, $\mathbf{I=0,\ldots,n_v}$
- dual magnetic gauge field: $\tilde{\mathbf{A}}_{\mu\mathbf{I}}$
- embedding tensor: $\mathbf{\Theta}_{\mathbf{\Lambda}}^{\lambda} = (\mathbf{\Theta}_{\mathbf{I}}^{\lambda}, \tilde{\mathbf{\Theta}}^{\mathbf{I}\lambda})$, $\mathbf{\Lambda=0,\ldots,2n_v}$

  $$D_\mu q = \partial_\mu q - \mathbf{A}_\mu^{\mathbf{I}}\, \mathbf{\Theta}_{\mathbf{I}}^{\lambda}\, k_\lambda(q) - \tilde{\mathbf{A}}_{\mu\mathbf{I}}\, \tilde{\mathbf{\Theta}}^{\mathbf{I}\lambda}\, k_\lambda(q) \,, \qquad k_\lambda \equiv \text{Killing vectors}$$

  $\mathbf{\Theta}_{\mathbf{\Lambda}}^{\lambda}$ is constrained by:

  - $N = 2$ supersymmetry
  - mutual locality of charges

- two-forms $B_{\mu\nu}$ with only topological couplings, algebraic field eqs. and appropriate gauge transformations to keep d.o.f. intact

$N = 2$ supersymmetry variations of the fermions

gravitino : $\delta_\epsilon \mathbf{\Psi}_{\mu\mathcal{A}} = D_\mu \epsilon_\mathcal{A} - \mathbf{S}_{\mathcal{A}\mathcal{B}} \gamma_\mu \epsilon^\mathcal{B} + \dots\,,$ $\quad {\scriptstyle \mathcal{A},\mathcal{B}=1,2}$

gaugino : $\delta_\epsilon \lambda^{\mathbf{i}\mathcal{A}} = \mathbf{W}^{\mathbf{i}\mathcal{A}\mathcal{B}} \epsilon_\mathcal{B} + \dots\,,$ $\quad {\scriptstyle i=1,\dots,n_\mathrm{v}}$

hyperino : $\delta_\epsilon \zeta_\alpha = \mathbf{N}_\alpha^\mathcal{A} \epsilon_\mathcal{A} + \dots\,,$ $\quad {\scriptstyle \alpha=1,\dots,2n_\mathrm{h}}$

where

$$\mathbf{S}_{\mathcal{A}\mathcal{B}} \sim \mathbf{V}^{\mathbf{\Lambda}} \mathbf{\Theta}_{\mathbf{\Lambda}}^{\lambda} \mathbf{P}_\lambda^{\mathbf{x}} \sigma^{\mathrm{x}}_{\mathcal{A}\mathcal{B}}\,, \qquad {\scriptstyle x=1,2,3}$$

$$\mathbf{W}^{\mathbf{i}\mathcal{A}\mathcal{B}} \sim (\nabla^i \bar{\mathbf{V}}^{\mathbf{\Lambda}})\, \mathbf{\Theta}_{\mathbf{\Lambda}}^{\lambda} \mathbf{P}_\lambda^{\mathbf{x}} \sigma^{\mathrm{x}\mathcal{A}\mathcal{B}}\,, \qquad {\scriptstyle I=0,\dots,n_\mathrm{v}}$$

$$\mathbf{N}_\alpha^\mathcal{A} \sim \bar{\mathbf{V}}^{\mathbf{\Lambda}} \mathbf{\Theta}_{\mathbf{\Lambda}}^{\lambda} U_{\alpha\mathrm{u}}^\mathcal{A} \mathbf{k}_\lambda^{\mathbf{u}}\,,$$

($U_{\alpha\mathrm{u}}^\mathcal{A}$ is vielbein of quaternionic metric)

Recall: $\mathbf{k}_\lambda \cdot \mathbf{J}^{\mathbf{x}} = \nabla \mathbf{P}_\lambda^{\mathbf{x}}\,, \qquad \mathbf{V}^{\mathbf{\Lambda}} = (\mathbf{X}^{\mathbf{I}}, \mathbf{F}_{\mathbf{I}})$

Spontaneous $N = 2 \to N = 1$ supersymmetry breaking

preserving $N = 1$ corresponds to

$$\begin{aligned} \langle \delta_{\epsilon_1} \lambda^{i\mathcal{A}} \rangle &= 0 \, , \quad \langle \delta_{\epsilon_1} \zeta_\alpha \rangle = 0 \, , \quad \langle \delta_{\epsilon_1} \Psi_{\mu\mathcal{A}} \rangle = 0 \\ \langle \delta_{\epsilon_2} \lambda^{i\mathcal{A}} \rangle &\neq 0 \, , \quad \langle \delta_{\epsilon_2} \zeta_\alpha \rangle \neq 0 \, , \quad \langle \delta_{\epsilon_2} \Psi_{\mu\mathcal{A}} \rangle \neq 0 \end{aligned}$$

which implies (dropped $\langle \ \rangle$)

$$\begin{aligned} \mathbf{W}_{\mathbf{i}\mathcal{A}\mathcal{B}} \, \epsilon_1^{\mathcal{B}} &= 0 \, , \qquad \mathbf{N}_{\alpha\mathcal{A}} \, \epsilon_1^{\mathcal{A}} = 0 \, , \qquad \mathbf{S}_{\mathcal{A}\mathcal{B}} \, \epsilon_1^{\mathcal{B}} = \tfrac{1}{2} \mu \epsilon^*_{1\,\mathcal{A}} \, , \\ \mathbf{W}_{\mathbf{i}\mathcal{A}\mathcal{B}} \, \epsilon_2^{\mathcal{B}} &\neq 0 \, , \qquad \mathbf{N}_{\alpha\mathcal{A}} \epsilon_2^{\mathcal{A}} \neq 0 \, , \qquad \mathbf{S}_{\mathcal{A}\mathcal{B}} \, \epsilon_2^{\mathcal{B}} \neq \tfrac{1}{2} \mu \epsilon^*_{2\,\mathcal{A}} \, , \end{aligned}$$

where

$$D_\mu \epsilon_{1\,\mathcal{A}} = \tfrac{1}{2} \mu \gamma_\mu \epsilon^*_{1\,\mathcal{A}} \, , \qquad \mu \sim \textbf{cosmological constant}$$

This implies:

$N = 1$ vacua are AdS or Minkowskian and automatically stable

Super-Higgs effect

➪ recall $N = 2$ spectrum:

$$\begin{array}{ll} \textbf{gravity multiplet} & \mathbf{s = [2, \frac{3}{2}, \frac{3}{2}, 1]} \\ \textbf{vector multiplet} & \mathbf{s = [1, \frac{1}{2}, \frac{1}{2}, 0, 0]} \\ \textbf{hyper multiplet} & \mathbf{s = [\frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0]} \end{array}$$

➪ reorganizes after partial breaking into $N = 1$ multiplets

$$\begin{array}{lll} \text{massive} & \textbf{gravitino multiplet} & \mathbf{s = [\frac{3}{2}, 1, 1, \frac{1}{2}]} \\ \text{massless} & \textbf{gravity multiplet} & \mathbf{s = [2, \frac{3}{2}]} \\ & \textbf{vector multiplets} & \mathbf{s = [1, \frac{1}{2}]} \\ & \textbf{chiral multiplets} & \mathbf{s = [\frac{1}{2}, 0, 0]} \end{array}$$

Super-Higgs effect

➪ recall $N = 2$ spectrum:

gravity multiplet $\mathbf{s} = [\mathbf{2}, \frac{\mathbf{3}}{\mathbf{2}}, \frac{\mathbf{3}}{\mathbf{2}}, \mathbf{1}]$

vector multiplet $\mathbf{s} = [\mathbf{1}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}}, \mathbf{0}, \mathbf{0}]$

hyper multiplet $\mathbf{s} = [\frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}]$

➪ reorganizes after partial breaking into $N = 1$ multiplets

massive **gravitino multiplet** $\mathbf{s} = [\frac{\mathbf{3}}{\mathbf{2}}, \mathbf{1}, \mathbf{1}, \frac{\mathbf{1}}{\mathbf{2}}]$

➪ minimal ingredients for partial supersymmetry breaking:

1. **one vector multiplet**
2. **one charged hyper** $\Rightarrow$ **2 Goldstone bosons**
3. **two commuting Killing vectors in hyper-sector**

gravitino & gaugino variations

vanishing gravitino & gaugino variations implies

$$\left(\Theta_I^{\lambda} - F_{IJ}\tilde{\Theta}^{J\lambda}\right) P_{\lambda}^{x}\, \sigma^{x}_{\mathcal{AB}}\epsilon_1^{\mathcal{B}} \;=\; 0 \;, \qquad \lambda=1,2$$

- ➪ "no" $N = 1$ solution for $\tilde{\Theta}^{J\lambda} = 0$ [Cecotti,Girardello,Porrati]
- ➪ two Killing vectors $k_{1,2} \Rightarrow$ choose convenient $SU(2)$-frame

$$P_1^3 = P_2^3 \equiv 0 \;, \qquad \text{and define} \quad P_{1,2}^{\pm} = P_{1,2}^1 \pm i\, P_{1,2}^2$$

- ➪ $N = 1$ solution in terms of complex vector C^I [Smyth,Triendl,JL]

$$\Theta_I^1 = -\,\mathrm{Im}(P_2^+\, F_{IJ}\, C^J) \;, \qquad \tilde{\Theta}^{I1} = -\mathrm{Im}(P_2^+\, C^I) \;,$$
$$\Theta_I^2 = \;\mathrm{Im}(P_1^+\, F_{IJ}\, C^J) \;, \qquad \tilde{\Theta}^{I2} = \;\mathrm{Im}(P_1^+\, C^I) \;,$$

- ➪ locality of charges implies $\bar{C}^I(\mathrm{Im}F)_{IJ}C^J = 0$

Note: no constraint on M_v^{SK}

hyperino variation

Recall: need 2 commuting Killing vectors $\mathbf{k_{1,2}}$ and

$$\mathbf{N}_{\alpha\mathcal{A}}\,\epsilon_1^{\mathcal{A}} = 0\ , \quad \mathbf{N}_{\alpha\mathcal{A}}\epsilon_2^{\mathcal{A}} \neq 0\ , \qquad \mathbf{N}_{\alpha}^{\mathcal{A}} \sim U_{\alpha\mathrm{u}}^{\mathcal{A}}\,\hat{\mathbf{k}}^{\mathbf{u}}\ , \quad \hat{\mathbf{k}}^{\mathbf{u}} \equiv \bar{\mathbf{V}}^{\mathbf{\Lambda}}\,\mathbf{\Theta}_{\mathbf{\Lambda}}^{\lambda}\,\mathbf{k}_{\lambda}^{\mathbf{u}}$$

this implies

$$\hat{\mathbf{k}} \cdot \mathbf{J}^{\mathbf{3}} = \mathrm{i}\ \hat{\mathbf{k}}\ , \qquad \hat{\mathbf{k}} \cdot (\mathbf{J}^{\mathbf{1}} - \mathbf{i}\mathbf{J}^{\mathbf{2}}) = 0$$

$\Rightarrow \mathbf{M}_{\mathbf{h}}^{\mathbf{QK}}$ is constrained by existence of holomorphic $\hat{\mathbf{k}}$

hyperino variation

Recall: need 2 commuting Killing vectors $\mathbf{k_{1,2}}$ and

$$\mathbf{N}_{\alpha\mathcal{A}}\,\epsilon_1^{\mathcal{A}} = 0\ , \quad \mathbf{N}_{\alpha\mathcal{A}}\epsilon_2^{\mathcal{A}} \neq 0\ , \qquad \mathbf{N}_{\alpha}^{\mathcal{A}} \sim U_{\alpha\mathrm{u}}^{\mathcal{A}}\,\hat{\mathbf{k}}^{\mathbf{u}}\ , \quad \hat{\mathbf{k}}^{\mathbf{u}} \equiv \bar{\mathbf{V}}^{\mathbf{\Lambda}}\,\mathbf{\Theta}_{\mathbf{\Lambda}}^{\lambda}\,\mathbf{k}_{\lambda}^{\mathbf{u}}$$

this implies

$$\hat{\mathbf{k}} \cdot \mathbf{J^3} = \mathrm{i}\ \hat{\mathbf{k}}\ , \qquad \hat{\mathbf{k}} \cdot (\mathbf{J^1} - \mathbf{iJ^2}) = 0$$

$\Rightarrow \mathbf{M_h^{QK}}$ is constrained by existence of holomorphic $\hat{\mathbf{k}}$

$N = 1$ moduli space

solution of:

$$\delta\mathbf{\Theta}_{\mathbf{\Lambda}}^{\lambda} = 0\ , \qquad \delta\big(\hat{\mathbf{k}} \cdot \mathbf{J^3} - \mathrm{i}\ \hat{\mathbf{k}}\big) = 0$$

generically stabilizes large number of moduli!

Outline for the rest of the talk

1. assume $\mathbf{M_h^{QK}}$ with holomorphic Killing vector exists and derive $N=1$ effective action

2. construct explicit solution for type II string theory

$N = 1$ effective action

steps:

- integrate out massive gravitino multiplet $(\mathbf{\frac{3}{2}}, \mathbf{1}, \mathbf{1}, \mathbf{\frac{1}{2}})$
  together with all multiplets at the same scale

- compute $\mathcal{L}_{\text{eff}}^{\mathbf{N=1}}(\mathbf{K}, \mathbf{W}, \mathbf{f})$ in terms of "$N = 2$ data"

- $N = 1$ constraints:
  - scalar field space is Kähler (with Kähler potentials $\mathbf{K}$)
  - $\mathbf{W}, \mathbf{f}$ are holomorphic

  (None of these constraints hold in $N = 2$.)

$N = 1$ scalar field space

Recall $N = 2$ scalar field space $\quad \mathbf{M} = \mathbf{M_v^{SK}(t)} \times \mathbf{M_h^{QK}(q)}$

integrating out massive multiplets takes:

- subspace $\quad \mathbf{M^{SK}_{N=1}} \subset \mathbf{M^{SK}_v}$ – automatically Kähler
- quotient $\quad \mathbf{M^{QK}_{N=1}} \subset \mathbf{M^{QK}_h}/(\mathbf{k_1}, \mathbf{k_2})$ [Cortés,Smyth,Triendl,JL]

  with Kähler metric $\mathbf{\hat{h}_{uv}}$ and Kählerform $\mathbf{\hat{K}}$

$$\mathbf{\hat{h}_{uv}} = \mathbf{h_{uv}} - \big(\mathbf{k_{1u}k_{1v}} + \mathbf{k_{2u}k_{2v}}\big)/\mathbf{k_1^2}\ , \qquad \mathbf{\hat{K}} = \mathbf{d}\omega^{\mathbf{3}}$$

  (Note: $\mathbf{M^{QK}_{N=1}}$ can be as large as $\underline{4n_{\rm h} - 2}$)

- resulting $N = 1$ scalar field space is indeed Kähler

$$\mathbf{M_{N=1}} \;=\; \mathbf{M^{QK}_{N=1}} \times \mathbf{M^{SK}_{N=1}}$$

$N = 1$ holomorphic couplings

- superpotential: $\mathbf{W} = \mathbf{e}^{\frac{1}{2}\mathbf{K}} \mathbf{V}^{\boldsymbol{\Lambda}} \boldsymbol{\Theta}_{\boldsymbol{\Lambda}}^{\lambda} \mathbf{P}_{\lambda}^{-}$, $\bar{\partial}\mathbf{W} = \mathbf{0}$

- gauge kinetic function: $\mathbf{f}_{\mathbf{IJ}} = \mathbf{F}_{\mathbf{IJ}}|_{\mathbf{N=1}}$, $\bar{\partial}\mathbf{f} = 0$

$N = 1$ solutions in type II string theory

➪ consider type II compactified on 6-dim. manifolds with $SU(3) \times SU(3)$-structure [Grana,Waldram,JL,...]

➪ $d = 4$ low energy effective action is an $N = 2$ supergravity

➪ $\mathbf{M_h^{QK}}$ is in the image of c-map [Cecotti,Girardello,Ferrara,Sabharwal]

This implies:

- $(2n_\mathrm{h} - 2)$-dim. special Kähler subspace with RR-scalars fibered over it
- $(2n_\mathrm{h} - 1)$ isometries associated with RR-scalars exist

➪ explicit $N = 1$ solution can be constructed

$N=1$ solutions in type II string theory

➪ $N=1$ solution in terms of "doubly symplectic" charge matrix

$$\Theta = \begin{pmatrix} \mathbf{e_{AI}} & \mathbf{p^{I}_{A}} \\ \mathbf{m^{A}_{I}} & \mathbf{q^{AI}} \end{pmatrix} , \qquad {\scriptstyle A=0,\ldots,n_{\rm h}-1, \quad I=0,\ldots,n_{\rm v}}$$

$$\text{with} \quad \mathbf{e_{AI}} = \mathrm{Re}(\bar{F}_{IJ}\bar{C}^{J}G_{AB}\mathbf{D^{B}}) , \qquad \mathbf{p^{I}_{A}} = (\bar{C}^{J}G_{AB}\mathbf{D^{B}}) ,$$

$$\mathbf{m^{A}_{I}} = \mathrm{Re}(\bar{F}_{IJ}\bar{C}^{J}\mathbf{D^{A}}) , \qquad \mathbf{q^{AI}} = \mathrm{Re}(\bar{C}^{I}\mathbf{D^{A}})$$

where $\quad \mathbf{\bar{D}^{A}}\mathrm{Im}(G_{AB})\mathbf{D^{B}} = 0,$

(G is prepotential of special Kähler base)

➪ Remarks:

- solution is mirror symmetric
- solution only exists for **non-geometric fluxes/torsion**
- analogous solution for AdS background

$N = 1$ solutions in type II string theory

- $N = 1$ solution in terms of "doubly symplectic" charge matrix

$$\Theta = \begin{pmatrix} \mathbf{e_{AI}} & \mathbf{p_A^I} \\ \mathbf{m_I^A} & \mathbf{q^{AI}} \end{pmatrix} , \qquad A=0,\ldots,n_{\mathrm{h}}-1, \quad I=0,\ldots,n_{\mathrm{v}}$$

with
$$\mathbf{e_{AI}} = \mathrm{Re}(\bar{F}_{IJ}\bar{C}^J G_{AB}\mathbf{D^B}) , \qquad \mathbf{p_A^I} = (\bar{C}^J G_{AB}\mathbf{D^B}) ,$$
$$\mathbf{m_I^A} = \mathrm{Re}(\bar{F}_{IJ}\bar{C}^J \mathbf{D^A}) , \qquad \mathbf{q^{AI}} = \mathrm{Re}(\bar{C}^I \mathbf{D^A})$$

where $\quad \mathbf{\bar{D}^A}\mathrm{Im}(G_{AB})\mathbf{D^B} = 0,$

(G is prepotential of special Kähler base)

- $N = 1$ Kähler potential:

$$K = K_{\mathrm{SK}} + 2\phi$$

can be expressed in terms of holomorphic coordinates given in

[Rocek,Vafa,Vandoren]

Conclusion

- $N = 2 \rightarrow N = 1$ generically possible if magnetic charges present
  – both in Minkowski and AdS backgrounds
- no constraint on $\mathbf{M_v^{SK}}$
- $\mathbf{M_h^{QK}}$ is constraint by existence of holomorphic $\mathbf{k}$
- $N = 1$ effective action can be constructed
  with Kählerian scalar field space and holomorphic $\mathbf{W}, \mathbf{f}$
- for type II compactified on manifolds with
  $SU(3) \times SU(3)$-structure $N = 1$ solutions exist
  if non-geometric fluxes/torsion are present