Constraints on bulk and defect correlators: the Maldacena-Wilson line in $\mathcal{N} = 4$ SYM

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Based on:
Work in progress 25xx.xxxx and 25xx.xxxx
with G. Bliard, B. Girault, J. Julius, M. Paulos, N. Suchel

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- $\mathcal{N} = 4$ SYM first example of AdS/CFT correspondence: dual to Type IIB string theory on $AdS_5 \times S^5$ [Maldacena '98].

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- Include extended objects: 1/2-BPS Maldacena-Wilson line $\rightarrow$ access to new observables.
- Unique interplay between localization, conformal bootstrap, integrability.
- Some constraints and techniques can be used for other defects as well.

The 1/2 BPS Wilson line in $\mathcal{N} = 4$ SYM

- $\mathcal{N} = 4$ SYM in $4d$, $SU(N)$ gauge group, integrable for $N \to \infty$.

$$S = \frac{1}{g^2} \mathrm{Tr} \int d^4x \Big(\frac{1}{2} F^{\mu\nu} F_{\mu\nu} + D^\mu \Phi^I D_\mu \Phi^I - \frac{1}{2} [\Phi^I, \Phi^J][\Phi^I, \Phi^J] + i\bar{\Psi} \not{D} \Psi + \bar{\Psi} \Gamma^I [\Phi^I, \Psi] + \partial_\mu \bar{c} D^\mu c + (\partial^\mu A_\mu)^2 \Big) , \quad \lambda = g^2 N$$

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- Introduce a 1/2-BPS Maldacena-Wilson line [Maldacena '98]

$$\mathcal{W}_\ell = \frac{1}{N} \mathrm{Tr}\, \mathcal{P} \exp \int_{-\infty}^{+\infty} d\tau \left(i\dot{x}^\mu A_\mu + |\dot{x}| \theta \cdot \Phi \right) ,$$

with expectation value $\langle \mathcal{W}_\ell \rangle = 1$ [Erickson, Semenoff, Zarembo '00].

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- Wilson line breaks symmetries:

$$\mathfrak{psu}(2,2|4) \to \mathfrak{osp}(4^*|4)$$
$$\mathfrak{so}(5,1) \times \mathfrak{so}_R(6) \to \mathfrak{so}(2,1) \times \mathfrak{so}(3) \times \mathfrak{so}_R(5)$$

Broken Ward identities

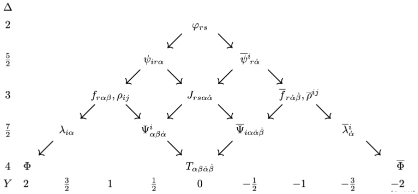

Figure: $\mathcal{O}_{20'}$ multiplet [Dolan, Osborn '01]

- Stress-tensor multiplet
  $\mathcal{O}_{20'} = \frac{1}{\sqrt{n_2}} \mathrm{Tr}[u^I u^J \Phi^I(x) \Phi^J(x)]$,
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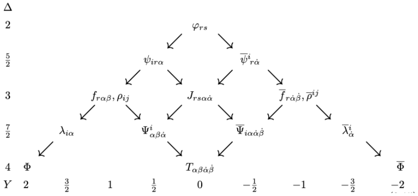

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$$\partial_\mu T^{\mu i}(y,\tau) = \delta^{(3)}(y) \underbrace{D^i(\tau)}_{\text{displacement}}, \qquad \partial^\mu \Psi^a_{\mu\dot{\alpha}}(y,\tau) = \delta^{(3)}(y) \underbrace{\psi^a_{\dot{\alpha}}(\tau)}_{\text{displacino}},$$

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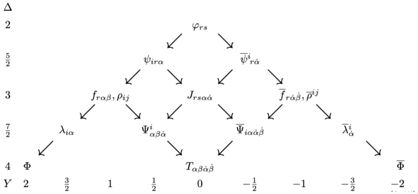

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- Combined into supermultiplet $\mathcal{D}_1 = \frac{1}{\sqrt{\hat n_1}}\mathcal{W}_\ell[v\cdot\hat\phi]$:

$$\mathcal{D}_1(\tau): \quad t^{\tilde a}(\tau) \to \psi^a_\alpha(\tau) \to D^i(\tau)$$

Defect four-point functions

- 4-pt correlator of displacement multiplet [Liendo, Meneghelli, Mitev '18]:

$$\langle \mathcal{D}_1(\tau_1)\mathcal{D}_1(\tau_2)\mathcal{D}_1(\tau_3)\mathcal{D}_1(\tau_4)\rangle = \frac{G(\chi)}{\tau_{12}^2\tau_{34}^2}\,, \quad \chi \equiv \frac{\tau_{12}\tau_{34}}{\tau_{13}\tau_{24}}\,, \quad \tau_{ij} = \tau_i - \tau_j\,,$$

$$G(\chi) = \mathbb{F}\chi^2 + (2\chi^{-1} - 1)f(\chi) - (\chi^2 - \chi + 1)f'(\chi)$$

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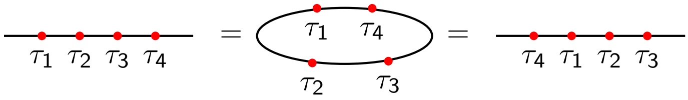
- 1d crossing symmetry for $f(\chi)$:



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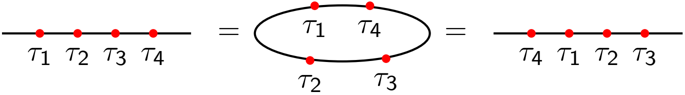

- Decomposition in superconformal blocks:

$$f(\chi) = F_{\mathcal{I}}(\chi) + \underbrace{\hat{c}^2_{\mathcal{D}_2}}_{\text{localization: [Giombi, Komatsu '18]}} F_{\mathcal{D}_2}(\chi) + \sum_{\hat{\Delta}} \hat{c}^2_{\hat{\Delta}} F_{\hat{\Delta}}(\chi)\,.$$

Conformal bootstrap + Integrability = Bootstrability

- Theory is integrable for $N \to \infty$. Use Quantum Spectral Curve (QSC) [Gromov, Kazakov, Leurent, Volin, '13 '14] to obtain spectrum:

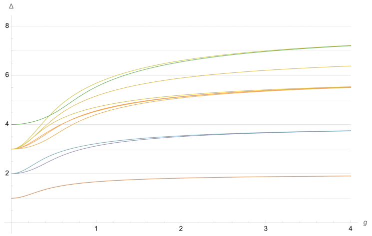

Figure: [Cavaglià, Gromov, Julius, Preti, '22]

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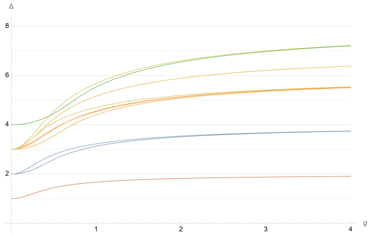

Figure: [Cavaglià, Gromov, Julius, Preti, '22]

- Add integrated correlators [Cavaglià, Gromov, Julius, Preti, '21 '22]

$$\int_0^1 dx \frac{1+\log\chi}{\chi^2}\delta G(\chi) = \frac{3\mathbb{C}-\mathbb{B}}{8\mathbb{B}^2}\,, \int_0^1 dx \frac{\delta f(\chi)}{\chi} = \frac{\mathbb{C}}{4\mathbb{B}^2} + \mathbb{F} - 3\,.$$

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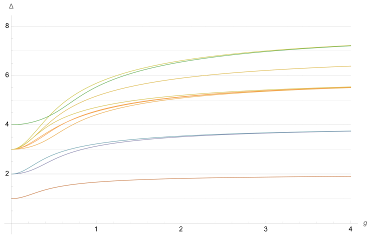

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- RHS can be computed exactly for any coupling g with integrability ($\mathbb{C}$) [Gromov, Levkovich-Maslyuk '15] or localization ($\mathbb{B}$) [Correa, Henn, Maldacena, Sever '12, Fiol, Garolera, Lewcowycz '12].

Bounds on OPE coefficients

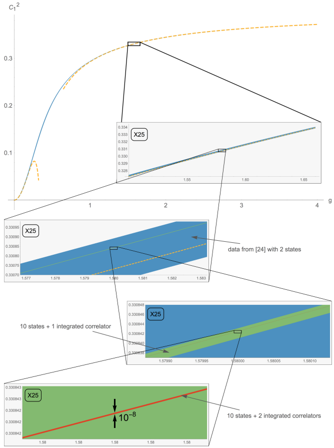

Figure: Bounds on OPE coefficient $\hat{c}_{\Delta_0}$ [Cavaglià, Gromov, Julius, Preti, '22]

Compared with analytic bootstrap results at

- Strong coupling [Ferrero, Meneghelli '21]
- Weak coupling [Cavaglià, Gromov, Julius, Preti, '22]

Bulk operators

- Bulk operators in the presence of the Wilson line: defect OPE (DOE)

$$\mathcal{O}_{\Delta}(y,\tau) \sim \sum_{\hat{\Delta}} \mu_{\Delta}^{\hat{\Delta}} C(y,\partial_\tau)\hat{\mathcal{O}}_{\hat{\Delta}}(\tau)\,.$$

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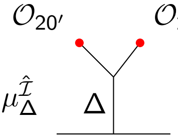

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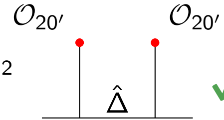

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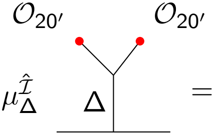
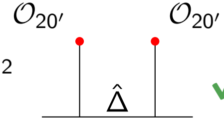
✗ $\sum_\Delta c_\Delta\, \mu_\Delta^{\hat{\mathcal{I}}}$ 

$= \sum_{\hat{\Delta}} (\mu_2^{\hat{\Delta}})^2$ 

✓

- Need to include 3-pt function for numerical conformal bootstrap:

$$\langle \mathcal{O}_{20'}(y_1,\tau_1)\mathcal{D}_1(\tau_2)\mathcal{D}_1(\tau_3)\rangle = \frac{G_{3\text{pt}}(z)}{y_1^2\,\tau_{23}^2}\,, \quad z = \frac{y_1^2\tau_{23}^2}{x_{12}^2\,x_{13}^2}\,,$$

$$G_{3\text{pt}}(z) = F_{\mathcal{I}}(z) + \mu_2^2\,\hat{c}_{\mathcal{D}_2} F_{\mathcal{D}_2}(z) + \sum_{\hat{\Delta}} \mu_2^{\hat{\Delta}}\,\hat{c}_{\hat{\Delta}} F_{\hat{\Delta}}(z)\,,$$

Bootstrapping bulk operators

Similar setup as [Ghosh, Paulos, Suchel '25]

1d crossing, integrated correlators

sum rules from broken WI

[Girault, Paulos, PvV WiP]

$$\begin{pmatrix} \hat{c}_{\hat{\Delta}} & \mu_2^{\hat{\Delta}} \end{pmatrix} \begin{pmatrix} \langle \mathcal{D}_1\mathcal{D}_1\mathcal{D}_1\mathcal{D}_1 \rangle & \langle \mathcal{O}_{20'}\mathcal{D}_1\mathcal{D}_1 \rangle \\ \langle \mathcal{O}_{20'}\mathcal{D}_1\mathcal{D}_1 \rangle & \langle \mathcal{O}_{20'}\mathcal{O}_{20'} \rangle \end{pmatrix} \begin{pmatrix} \hat{c}_{\hat{\Delta}} \\ \mu_2^{\hat{\Delta}} \end{pmatrix}$$

locality

[Levine, Paulos '23]

NO crossing, integrated correlators

[Billò, Galvagno, Frau, Lerda '23]

+ Quantum Spectral Curve for spectrum.

Deriving the sum rules

- Defect breaks symmetry: generators split into broken and unbroken:

$$[Q^{\tilde{a}}, Q^{b}] = f^{\tilde{a}b\tilde{c}} Q^{\tilde{c}} \,, \quad [Q^{\tilde{a}}, Q^{\tilde{b}}] = f^{\tilde{a}\tilde{b}c} Q^{c} + f^{\tilde{a}\tilde{b}\tilde{c}} Q^{\tilde{c}} \,,$$
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- Use broken Ward identities (e.g. for the tilt):

$$Q^{\tilde{a}} \left|0\right\rangle_{D} = \int d^{d}x \, \partial_{\mu} J^{\tilde{a},\mu} \left|0\right\rangle_{D} = \int d^{p}\tau \, t^{\tilde{a}} \left|0\right\rangle_{D} \,,$$

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- Take correlators with bulk insertions $\mathcal{X} = \mathcal{O}_1(x_1) \ldots \mathcal{O}_i(x_i)$

$$\langle \mathcal{Q}^{\tilde{a}} \mathcal{X} \rangle = \langle \delta_{\tilde{a}} \mathcal{X} \rangle \,.$$

Deriving the sum rules

- For insertion of $Q^{\tilde{a}}$ at time T, close contour to past or future:

$$-\int_{x_0>T} d^p\tau \langle t^{\tilde{a}}(\tau)\mathcal{X}\rangle = \int_{x_0<T} d^p\tau \langle t^{\tilde{a}}(\tau)\mathcal{X}\rangle + \langle \delta_{\tilde{a}}\mathcal{X}\rangle \,,$$

- resulting in

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- Directly applicable to bulk-defect 3pt function (set $y_2 \to 0$):

$$\int d\tau_3 \, \langle \mathcal{O}_{20'}(y_1,\tau_1)\mathcal{D}_1(\tau_2)\mathcal{D}_1(\tau_3)\rangle = \frac{C(N,\lambda)}{y_1^2+\tau_{12}^2} \times h(u_1,v_2,v_3) \,,$$

$C(N,\lambda) \sim \langle \mathcal{O}_{20'}(y_1\tau_1)\mathcal{D}_1(\tau_2)\rangle$ known from localization .

Deriving the sum rules

- Same idea for displacement (conformal generators $\mathcal{P}, \mathcal{K}, \mathcal{M}$) and displacino (SUSY generators $\mathcal{Q}_{a\alpha}, \mathcal{S}^{a\alpha}$). Find one sum rule for $\mathcal{D}_1$.

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- Check with perturbative computations for $\langle \mathcal{O}_{20'} \mathcal{D}_1 \mathcal{D}_1 \rangle$ [Artico, Barrat, Xu '24] and $\langle J^a_{\mu\dot{\alpha}} \mathcal{D}_1 \mathcal{D}_1 \rangle$ [Bliard, Julius, Paulos, Suchel, PvV WiP]

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- Could also start with $Q^{\tilde{a}}$ + three bulk operators: sum rule on $\langle \mathcal{D}_1 \mathcal{D}_1 \mathcal{D}_1 \mathcal{D}_1 \rangle$. Leads to known constraint on topological sector:

$$\mathbb{F} = 1 + \hat{c}^2_{\mathcal{D}_2} \, .$$

Back to integrated correlator constraints

- What if we insert two charges?

$$\int_{\tau_0 < T} \int_{\tau_0 > T} d\tau \, d\tau' \, \langle \mathcal{O}(\tau_1) [Q^{\tilde{a}}(\tau), Q^{\tilde{b}}(\tau')] \mathcal{O}(\tau_2) \rangle$$
$$= f^{\tilde{a}\tilde{b}c} \langle \mathcal{O}(\tau_1) \delta^c \mathcal{O}(\tau_2) \rangle + \dots$$

- Proportional to normalization Λ of tilt

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- For MW line: derived in [Drukker, Kong, Sakkas '22]. Found to be linear combination of two integrated constraints!
- Now independent of curvature function $\mathbb{C}$ [Cavaglià, Gromov, Julius, Preti '22]

Extensions and generalizations

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- Some already known:
  - 4pt tilt [Kutasov '89, Friedan, Konechy '12, Behan '17, Drukker, Kong, Sakkas '22]
  - 4pt displacement in 1d [Gabai, Sever, Zhong '25]
  - 2pt bulk-defect displacement [Billò, Goncalves, Lauria, Meineri '16]

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  - 4pt displacement in 1d [Gabai, Sever, Zhong '25]
  - 2pt bulk-defect displacement [Billò, Goncalves, Lauria, Meineri '16]
- 3pt bulk-defect, 4pt displacino, 4pt displacement in any d are new [Girault, Paulos, PvV WiP].

Extensions and generalizations

- Sum rules for general defects breaking conformal, global, or supersymmetry $\rightarrow$ individual constraints on correlators with t, D, ψ.
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- 3pt bulk-defect, 4pt displacino, 4pt displacement in any d are new [Girault, Paulos, PvV WiP].
- Combine with analytic functionals [Mazac, Paulos '18] or local blocks [Levine, Paulos '23] to obtain ready-to-use sum rules.

Conclusion and further outlook

- After success of bootstrability for (protected) defect multiplets, can now include bulk operators.
- Crossing is not enough/not present: rely on integrated correlators (+ locality).
- Reconstruct 2-pt function $\langle \mathcal{O}_{20'} \mathcal{O}_{20'} \rangle$ for any λ.

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- Include long multiplets as external operators?
- Constrain other defects: pinning line defect in $O(N)$ CFT [Gabia, '25, Girault, Paulos, PvV WiP] , non-SUSY deformation of Wilson line, ...

Thank you!

Backup

Inserting defect operators

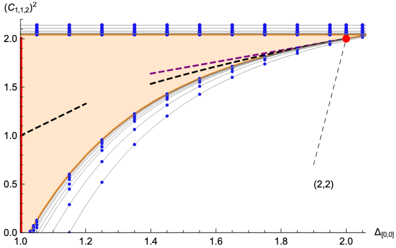
To insert operators on defect, we write:

$$\mathcal{D}_1 = \frac{1}{\sqrt{\hat{n}_1}} \mathcal{W}_\ell[v \cdot \hat{\phi}] = \frac{1}{N} \operatorname{Tr} \mathcal{P}[v \cdot \hat{\phi} \exp \int d\tau \, (iA_0 + \Phi^6)] \, .$$

Correlation functions are then written as

$$\begin{aligned} &\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_i(x_i) \hat{\mathcal{O}}_{i+1}(\tau_{i+1}) \dots \hat{\mathcal{O}}_{i+j}(\tau_{i+j}) \rangle \\ &= \langle \mathcal{O}_1(x_1) \dots \mathcal{O}_i(x_i) \mathcal{W}_\ell[\hat{\mathcal{O}}_{i+1}(\tau_{i+1}) \dots \hat{\mathcal{O}}_{i+j}(\tau_{i+j})] \rangle \, . \end{aligned}$$

Bootstrap for finite N



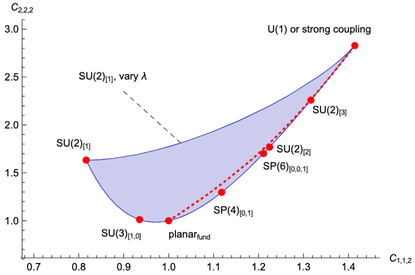


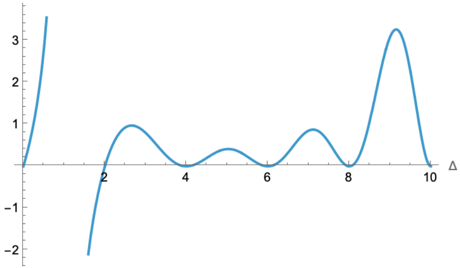
Figure: [Liendo, Meneghelli, Mitev '16]

For $\mathcal{N} = 4$ SYM without a defect, see e.g. [Chester, Dempsey, Pufu '21]

Sum rules using functionals

Use analytic functionals to perform integration of four-point correlator constraints. E.g. four tilts, single integration:

$$IC[\Delta] = \frac{4^{\Delta}\sin^2\left(\frac{\pi\Delta}{2}\right)\Gamma\left(\Delta+\frac{1}{2}\right)}{3\sqrt{\pi}(\Delta-2)(\Delta-1)\Gamma(\Delta+2)}\Big(-2(\Delta-1)\Delta$$
$$+(\Delta-2)(\Delta-1)(\Delta+1)\Delta\Big(\psi^{(1)}\left(\frac{\Delta}{2}\right)-\psi^{(1)}\left(\frac{\Delta+1}{2}\right)\Big)+6\Big)\,.$$



Sum rules with local blocks

For bulk-defect 3pt function, we take the local blocks from [Levine, Paulos '23]

$$
\begin{aligned}
IC_{3pt}[\Delta] = & \frac{\sin\left(\frac{\pi\Delta}{2}\right)\Gamma\left(\frac{\Delta}{2}-1\right)\Gamma\left(\Delta+\frac{5}{2}\right)}{2\rho\Gamma\left(\frac{\Delta}{2}+1\right)^2\Gamma\left(\frac{\Delta+5}{2}\right)} \\
& \times\left(\frac{4\rho\,{}_3F_2\left(\frac{1}{2},2,2;2-\frac{\Delta}{2},\frac{\Delta}{2}+\frac{7}{2};1\right)}{\Delta+5}+\frac{4-2\Delta}{\Delta^2+3\Delta+2}\right) \\
& +\frac{\sqrt{\pi}\Gamma\left(\frac{\Delta-1}{2}\right)\,{}_3F_2\left(\frac{\Delta}{2}-\frac{1}{2},\frac{\Delta}{2}+1,\frac{\Delta}{2}+1;\frac{\Delta}{2},\Delta+\frac{5}{2};1\right)}{\Gamma\left(\frac{\Delta}{2}\right)}\,.
\end{aligned}
$$