Scale separation on $AdS_3 \times S^3$ with and without supersymmetry

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Motivations

Salam-Sezgin model [Salam, Sezgin, 0370-2693(84)90589-6, P.L.B.]

$$\mathcal{L}_{(6d)} = R * \mathbb{1} - \frac{1}{4} * d\phi \wedge d\phi - \frac{1}{2} e^{\phi} * H \wedge H - \frac{1}{2} e^{\frac{1}{2}\phi} * F \wedge F - 8g^2 e^{-\frac{1}{2}\phi} * \mathbb{1}$$

Interesting features

- Solution of the type $(\mathrm{Minkowski})_4 \times S^2$
- Embedding in string theory still mysterious
- Elusive holographic CFT dual

2-parameter solution of the form $\mathrm{AdS}_3 \times \left(\text{squashed } \mathrm{S}^3\right)$ [Ma, Pang, Lü, JHEP04(2024)052]

$$ds^2 = \cosh^2\alpha \, \cosh^2\beta \, ds^2_{\mathrm{AdS}_3} + \frac{1}{4}\sigma_3^2 + \frac{1}{4}\cosh^2\beta \left(\sigma_1^2 + \sigma_2^2\right)$$

α ↙ R-symmetry gauging in 6D

β ↘ Squashing parameter of S^3

For $\alpha = \beta$ the solution is supersymmetric

Spectrum

Compute spectrum around the solution as a function of α and β (ExFT techniques) [A.P., Samtleben, Sezgin, 2504.12425, P.R.D]

$$\mathrm{OSp}(2|2) \otimes \mathrm{SL}(2,\mathbb{R})_R$$

$$\mathrm{Spec}_n = \mathrm{Spec}_{\mathrm{long},n} \oplus \mathrm{Spec}_{\mathrm{short},n}$$

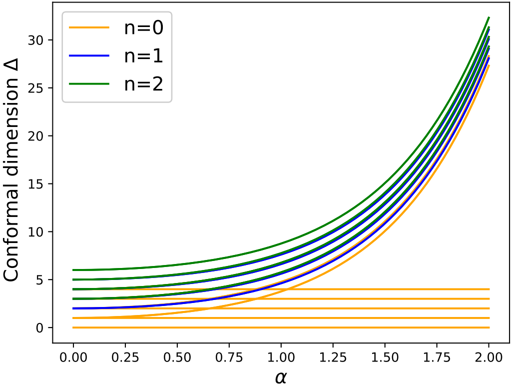
$$\begin{aligned}
\mathrm{Spec}_{\mathrm{long},n} \quad = \quad & \bigoplus_{u\in\mathcal{P}_{n-2}} \{[\tfrac{n}{2}]_u\,(0|-1+\Gamma_{n,u})\,\}_{\mathrm{L}} \oplus \bigoplus_{u\in\mathcal{P}_{n}} \{[\tfrac{n}{2}]_u\,(-1|\Gamma_{n,u})\,\}_{\mathrm{L}} \oplus \bigoplus_{u\in\mathcal{P}_{n-2}} \{[\tfrac{n}{2}]_u\,(2|1+\Gamma_{n,u})\,\}_{\mathrm{L}} \\
& \oplus \bigoplus_{u\in\mathcal{P}_{n}} \{[\tfrac{n}{2}]_u\,(1|2+\Gamma_{n,u})\,\}_{\mathrm{L}} \oplus \bigoplus_{u\in\mathcal{P}_{n-2}} \{[\tfrac{n\pm2}{2}]_u\,(1|\Gamma_{n,u})\,\}_{\mathrm{L}} \oplus \bigoplus_{u\in\mathcal{P}_{n}} \{[\tfrac{n\pm2}{2}]_u\,(0|1+\Gamma_{n,u})\,\}_{\mathrm{L}} \\
& \oplus \bigoplus_{u\in\mathcal{P}_{n-2}} \{[\tfrac{n}{2}]_u\,(1|\Gamma_{n,u})\,\}_{\mathrm{L}} \oplus \bigoplus_{u\in\mathcal{P}_{n}} \{[\tfrac{n}{2}]_u\,(0|1+\Gamma_{n,u})\,\}_{\mathrm{L}} \\
\mathrm{Spec}_{\mathrm{short},n} \quad = \quad & \{[\tfrac{n}{2}]_{\pm n}(0)\}_{\mathrm{S}} \;\oplus\; \{[\tfrac{n}{2}]_{\pm(n+2)}(-1)\}_{\mathrm{S}} \;\oplus\; \{[\tfrac{n}{2}]_{\pm n}(2)\}_{\mathrm{S}} \;\oplus\; \{[\tfrac{n}{2}]_{\pm(n+2)}(1)\}_{\mathrm{S}}
\end{aligned}$$

$$\Gamma_{n,u}(\alpha,\beta) = \sqrt{1+\cosh^2\alpha\,\big(n(n+2)+u^2\sinh^2\beta\big)}$$

- $\alpha \neq \beta$, supersymmetry completely broken

Scale separation

Conformal dimension in the supersymmetric case $\alpha = \beta$



Integer conformal dimension

$$\frac{\ell_{KK}}{\ell} \sim \frac{1}{\cosh \alpha \sinh \beta} \ll 1 \quad \text{for} \quad \alpha, \beta \gg 1$$