Seiberg-Witten geometry of $\widehat{D}$-type Little Strings

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based on [2407.11164] in collaboration with Stefan Hohenegger and Taro Kimura

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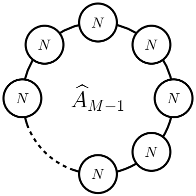
Little String Theories

Little String Theories: 6-dimensional theories, string-like degrees of freedom, doubly elliptic structure

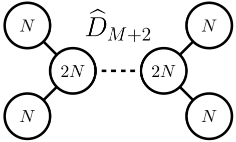
	$\mathbb{S}^1_0$	$\mathbb{S}^1_1$		$\mathbb{R}^3_{\parallel}$		$\mathbb{S}^1_5$	$\mathbb{S}^1_6$		$\mathbb{R}^4_{\perp}/\Gamma$		
	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}
$N\ M5$	$=$	$=$	$=$	$=$	$=$	$=$	$\times$				

Ex: $\Gamma = \mathbb{Z}_M \ \rightarrow \widehat{A}$-type



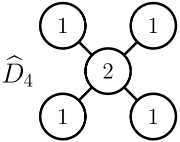
- Very well studied
- Partition function and Seiberg-Witten curve known for generic N,M

Ex: $\Gamma = \text{Dih}_{2M} \ \rightarrow \widehat{D}$-type



- Much less studied
- Seiberg-Witten curve only for $M = 2$, no general expression for the partition function

Doubly elliptic Seiberg-Witten curves



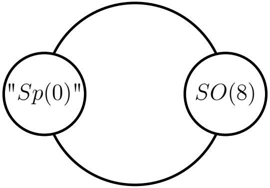
– $(\widehat{D}_4,1)$ Little String Theory:

$$\theta_3^2(0;\rho)\theta_2^2(0;\rho)\theta_1^2(z_2;\rho)c_1(\tau)\prod_{i=1}^{4}\theta_1(z_1-a_i;\tau)\theta_1(z_1+a_i;\tau)-\theta_4^2(z_2;\rho)\theta_1^2(2z_1;\tau)=0$$

[Haghighat, Kim, Yan, Yau; Haghighat, Sun '18]

- $6=h^\vee(\widehat{D}_4)$ independent parameters $\rightarrow\{a_1,\ldots,a_4,\tau,\rho\}$
- invariant under $\mathbb{Z}_2$: $z_1\rightarrow -z_1$, $z_2\rightarrow -z_2$
- No mass deformation, genus 1 $\times$ genus 1 theta functions instead of genus 2 theta function
- dual to $SO(8)$ – "$Sp(0)$" $\widehat{A}_1$ quiver gauge theory

$\longrightarrow$ Generalisation beyond $\widehat{D}_4$?



Systematic approach to $(\widehat{D}_M, 1)$

Constraints:

- Invariant under $z_1 \to -z_1$ and $z_2 \to -z_2$
- $\sum_{1\leq i,j\leq N} \widetilde{T}_i(z_2;\rho)\mathbb{M}_{ij}(\tau)\Phi_j(z_1;\tau) = 0$, $\widetilde{T}_i$ encode information about the gauge group, $\mathbb{M}_{ij}\Phi_j$ encode information about the quiver structure [Haghighat, Yan, Yau '17]
- Single $M5$-brane states fractionalise due to the orbifold $\to$ $(2, 2M)$-polarised theta functions can be decomposed as the product of two $(1, M)$-polarised theta functions
- $2M - 2 = h^\vee(\widehat{D}_M)$ complex dimensional moduli space

Results

- Unique Seiberg-Witten curve matching lower dimensional theories for any M
- Solutions are characterised by the matrix $\mathbb{M}(\tau)$
- $\mathbb{M}(\tau)$ transforms simply under $\Gamma_0(M)$ modular subgroup
- Formulation in the dual $SO(2M) - Sp(M-4)$ setup

Example of solution for $\widehat{D}_5$:

$$\mathbb{M}_{\widehat{D}_5}(\tau) = \begin{bmatrix} \phi_0 & 0 & 0 & 0 & 0 & \phi_5 \\ 0 & \phi_5 & 0 & 0 & \phi_0 & 0 \\ -\phi_2 & 0 & \phi_0 & \phi_5 & 0 & -\phi_3 \\ 0 & -\phi_3 & \phi_4 & \phi_1 & -\phi_2 & 0 \\ 0 & \phi_1 & 0 & 0 & \phi_4 & 0 \\ \phi_4 & 0 & 0 & 0 & 0 & \phi_1 \end{bmatrix}$$

Outlook

- Explore the range of accessible lower dimensional theories
- Connection to integrable systems (for A-type LSTs [BF,Hohenegger,Kimura '24])
- Extend our approach to E-type little string theories

Thank you for your attention!