An Unusual BPS Equation

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Tours - June 5, 2025

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2501.13197 (Costas Bachas, Lorenzo Bianchi, ZC)

Plan

- 1. What is it?
  background in defect CFT
- 2. Why is it unusual?
  gravitational perspective: rephrasing the BPS equation as $\sigma_{\text{grav}} = \sigma_{\text{iner}}$
- 3. Why is it interesting?
  CFT perspective: radiation of an accelerating charge
- 4. How to prove it?
  playing with superalgebras and superconformal Ward identities

Introduction

- Context: (rotation-invariant) conformal defects.

$$\mathrm{SO}(d,2) \to \mathrm{SO}(p,2) \times \mathrm{SO}(d-p)\ . \tag{1}$$

- (i) $\langle T^{\mu\nu} \rangle$ (energy-momentum stored in the fields)

$$\begin{aligned} \langle T^{\alpha\beta} \rangle &= a_T \left(\frac{d-p-1}{d}\right) \frac{\delta^{\alpha\beta}}{|x_\perp|^d}\ , \qquad \langle T^{\alpha i} \rangle = 0\ , \\ \langle T^{ij} \rangle &= a_T \left[\frac{x^i x^j}{|x_\perp|^{d+2}} - \frac{p+1}{d} \frac{\delta^{ij}}{|x_\perp|^d} \right]\ . \end{aligned} \tag{2}$$

(Billò, Gonçalves, Lauria, Meineri '16)

- (ii) $\langle D^i D^j \rangle$ (resistance to deformation)

$$\partial_\mu T^{\mu i} = \delta^{(d-p)}(x_\perp) D^i\ , \qquad \langle D^i(0) D^j(x_\parallel) \rangle = \frac{C_D\, \delta^{ij}}{|x_\parallel|^{2p+2}}\ . \tag{3}$$

- Note that a_T is not defined for boundary ($p = d-1$), and C_D is not defined for local operators ($p = 0$).
- In terms of C_D and a_T, our BPS equation reads,

$$\boxed{\frac{C_D}{a_T} = -\frac{2(d-1)(p+2)\Gamma(p+1)}{d\, \pi^{p-d/2} \Gamma(\frac{p}{2}+1) \Gamma(\frac{d-p}{2})}\ .} \tag{4}$$

(Lewkowycz, Maldacena '13; Bianchi, Lemos '19)

Gravitational and Inertial Tension

- In AdS/CFT, a p-dimensional defect is the boundary anchor of a p-brane.
- Local operators ($p = 0$) are endpoints of particle worldlines. For large operators, the dual is a black hole, whose mass is not locally defined. The ADM mass resums the classical GR corrections.
- The dilatation charge Δ resums both the classical GR corrections (in $G_\mathrm{N} m_0$) and the quantum fluctuations (in $1/m_0$). For example, in unit-radius AdS_4,

$$\begin{aligned}\Delta &= \frac{3}{2} + \sqrt{m_0^2 + \frac{9}{4}} + \text{GR corrections} \\ &= m_0 \left[1 + \frac{3}{2m_0} + \frac{9}{8m_0^2} + \cdots + G_\mathrm{N} m_0 + (G_\mathrm{N} m_0)^2 + \cdots \right] .\end{aligned} \tag{5}$$

- Can we define a similar invariant tension for p-branes?

Gravitational and Inertial Tension

- As an generalization of the dilatation charge, we define the **gravitational tension** as the integration of the dilatation current around the dual defect in CFT,

$$\sigma_{\text{grav}} \propto \oint \mathrm{d}s^i \, x^\mu \langle T_{\mu j} \rangle \,, \qquad \sigma_{\text{grav}} = \gamma_{\text{grav}} \cdot a_T \,. \tag{6}$$

- However, there is also another natural measure of tension in terms of its stiffness, which we call **inertial tension**,

$$\sigma_{\text{iner}} = \gamma_{\text{iner}} \cdot C_D \,. \tag{7}$$

- The pre-factors are fixed by going to the classical probe brane limit and demand that both σ reduce to the bare Nambu–Goto tension. One finds the following results by computing the corresponding Witten diagrams,

$$\gamma_{\text{grav}} = -\frac{2(d-1)\pi^{(d-p)/2}}{d\,\Gamma(\frac{d-p}{2})} \,, \qquad \gamma_{\text{iner}} = \frac{\pi^{p/2}\Gamma(\frac{p}{2}+1)}{(p+2)\Gamma(p+1)} \,. \tag{8}$$

- There is one subtlety in the calculation. There is no global Fefferman–Graham coordinates for both the bulk and the brane fields. In our case, absolute normalizations matter.
- To settle this, we calculate the correlator $\langle T^{\mu\nu} D^i \rangle$ using Witten diagrams. Schematically, the conformal Ward identities are of the form $\langle D^i \int_\perp (\partial_\mu T^{\mu j}) \rangle \sim \langle D^i D^j \rangle$ and $\langle T^{\mu\nu} \int_\parallel D^i \rangle \sim \partial^i \langle T^{\mu\nu} \rangle$. This fixes the ambiguity.

(Bachas, ZC '24)

- Our BPS equation reduces to

$$\sigma_{\text{grav}} = \sigma_{\text{iner}} \ . \tag{9}$$

Note that both sides receive different non-trivial corrections beyond the Nambu–Goto limit, however supersymmetry ensures that remain equal.

- For example, the F-string in $\mathrm{AdS}_5 \times S^5$ is dual to a line defect in $\mathcal{N} = 4$ SYM,

$$\begin{aligned} C_D &= -18 a_T = \frac{6}{\pi^2} \lambda \partial_\lambda \log\left(\frac{1}{N} \mathrm{e}^{\lambda/8N} L^1_{N-1}(-\frac{\lambda}{4N}) \right) , \\ \sigma_{\text{iner}} &= \sigma_{\text{grav}} = \frac{\sqrt{\lambda}}{2\pi} \left[1 + O(\frac{1}{\sqrt{\lambda}}) + O(\frac{\sqrt{\lambda}}{N}) + O(\frac{\sqrt{\lambda}}{N^2}) \right] , \end{aligned} \tag{10}$$

with $\lambda = g^2_{\text{YM}} N$. (Erickson, Semenoff, Zarembo ’00; Correa, Henn, Maldacena, Sever ’12)

- Usually a BPS equation relates mass (or tension) to charge. This unusual one relates two tensions.

- The origin of the conjecture: radiation of an accelerating quark in $\mathcal{N} = 4$ SYM.

  (Lewkowycz, Maldacena '13)

  The authors attempt to reconcile the discrepancy between two calculations of radiation: (i) from a kink ($\propto C_D$); (ii) from a uniformly accelerating quark ($\propto a_T$). It originates from the difficulty in separating the radiation from self-energy.
- They introduce the modified stress tensor, which is conserved but not traceless,

$$\widetilde{T}^{\mu\nu} = T^{\mu\nu} + \xi(\eta^{\mu\nu}\Box - \partial^\mu\partial^\nu)O \, . \tag{11}$$

$$\langle O(x)\rangle = \frac{a_O}{|x_\perp|^{d-2}} \implies \langle \widetilde{T}^{\mu\nu}\rangle = \langle T^{\mu\nu}\rangle + (\eta^{\mu\nu}\Box - \partial^\mu\partial^\nu)\frac{\xi a_O}{|x_\perp|^{d-2}} \, . \tag{12}$$

- Recall that the transverse stress is

$$\langle T^{ij}\rangle = a_T\left[\frac{x^i x^j}{|x_\perp|^{d+2}} - \frac{p+1}{d}\frac{\delta^{ij}}{|x_\perp|^d}\right] . \tag{13}$$

  It is possible to remove the transverse stress of the static field by choosing

$$\xi_{\text{stress}} = \frac{a_T}{d(d-2)a_O} \implies \langle \widetilde{T}^{ij}\rangle = 0 \, . \tag{14}$$

- This can be done without assuming supersymmetry. We will argue now that supersymmetry ensures the NEC-violating radiation is also removed.

- Example: Wilson line in 4d $\mathcal{N}=2$ super-QED (half-BPS iff $e=\pm g$),

$$W = \exp\left(\int \mathrm{d}s \ \mathrm{i}e\, A_\mu \dot{y}^\mu + g\,|\dot{y}|\phi\right) . \tag{15}$$

$$\begin{aligned} T^{(s)}_{\mu\nu} &= \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\eta_{\mu\nu}|\partial\phi|^2 + \frac{1}{6}(\eta^{\mu\nu}\Box - \partial^\mu\partial^\nu)\phi^2 \ , \\ T^{(v)}_{\mu\nu} &= F_\mu{}^\rho F_{\nu\rho} - \frac{1}{4}\eta_{\mu\nu}|F|^2 \ , \qquad D^i = eF^{0i} + g\partial^i\phi \ . \end{aligned} \tag{16}$$

- The scalar operator is $O=\phi^2$ and $a_O = g^2/16\pi^2$,

$$\xi_{\text{stress}} = \frac{a_T}{8a_O} = -\frac{g^2+3e^2}{24g^2} \overset{e=\pm g}{=\!=\!=} -\frac{1}{6} \ . \tag{17}$$

In the BPS case, this improvement also removes the $R\phi^2$ contribution which violates NEC (since $k_\mu k_\nu \partial^\mu\partial^\nu\phi^2$ does not have a definite sign).

(Fiol, Martínez-Montoya '19)

Radiation of Accelerating Charges

- More generally, to see the origin of the NEC-violating radiation, let us consider the energy flux in the background of a moving defect,

$$\langle T^{0i}(x) e^{\int y^j D^j} \rangle = \langle T^{0i}(x) \rangle + \int d\tau \; y^j(\tau) \langle T^{0i}(x) D^j(y) \rangle + \cdots \; . \tag{18}$$

- The $\langle TD \rangle$ term has singularities at both $|x_\perp| = 0$ and $|x - y| = 0$. The kinematic structure is fixed by conformal symmetry and we are left with three constants b_i,

$$\langle T^{0i}(x) D^j(y) \rangle = \frac{(x-y)^0}{|x_\perp|^{d-p} |x-y|^{2p+4}} \left(4b_1 \frac{x^i x^j |x_\perp|^2}{|x-y|^2} - b_3 \delta^{ij} |x_\perp|^2 + (b_3 - 2b_1) x^i x^j \right) . \tag{19}$$

- The b_i's are fixed in terms of C_D and a_T, since the $\langle TD \rangle$ correlator is related to $\langle T \rangle$ and $\langle DD \rangle$ by conformal Ward identities, and we have the traceless constraint for T,

$$\begin{gathered} (p+1) b_2 + b_1 = \frac{d}{2} b_3 \; , \qquad b_3 = 2^{p+2} \pi^{-(p+1)/2} \Gamma(\frac{p+3}{2}) a_T \; , \\ 2p\, b_2 - (2d - p - 2) b_3 = \frac{(d-p)\Gamma(\frac{d-p}{2})}{\pi^{(d-p)/2}} C_D \; . \end{gathered} \tag{20}$$

- We can write our BPS equation in the b_i parameters,

$$\boxed{\frac{b_1}{b_3} = \frac{p+2}{2d} \; .} \tag{21}$$

- The NEC-violating radiation comes from the leading lightcone singularity

$$\langle T^{0i}(x) D^j(y) \rangle = -\frac{32 b_1 |x_\perp|^{p+2-d}}{p(p+1)(p+2)} \partial^0 \partial^i \partial^j \left(\frac{1}{|x-y|^{2p}}\right) + \cdots \ . \tag{22}$$

- As an example, let us consider line defects in $d = 4$. We can replace the Euclidean propagator by the retarded propagator,

$$\frac{1}{4\pi |x-y|^2} \mapsto \frac{\mathrm{i}\theta(x^0 - y^0)}{2\pi} \delta(|x-y|^2) \ . \tag{23}$$

  Upon integration by parts for $\int \mathrm{d}\tau \ y^j(\tau) \langle T^{0i} D^j \rangle$, we get a triple τ-derivative acting on the displacement profile $y(\tau)$ (c.f. the derivation of the Liénard–Wiechert potential). The resulting term is proportional to $\partial_\tau^3 y = \dot{a}$. It has no definite sign (unlike a^2), hence violates the NEC. (Fiol, Martínez-Montoya '19)

- We can choose an appropriate value of $\xi = \xi_{\text{NEC}}$ to cancel the NEC-violating term (proportional to b_1),

$$\langle O(x) D^j(y) \rangle = b_O \frac{x_j |x_\perp|^{p-d+2}}{|x-y|^{2(p+1)}} \implies \xi_{\text{NEC}} = \frac{b_1}{(p+1)(p+2) b_O} , \quad b_O = 2^p \Gamma(\frac{p+1}{2}) \pi^{-(p+1)/2} (d-2) a_O . \tag{24}$$

- On the other hand, the value ξ_{stress} for the removal of the transverse stress is proportional to $b_3 \sim a_T$,

$$\xi_{\text{stress}} = \frac{a_T}{d(d-2) a_O} = \frac{b_3}{2d(p+1) b_O} . \tag{25}$$

- The two values of ξ are the same iff our BPS equation holds,

$$\boxed{\xi_{\text{NEC}} = \xi_{\text{stress}} \iff \frac{b_1}{b_3} = \frac{p+2}{2d} .} \tag{26}$$

Conclusion: supersymmetry ensures that when the transverse stress is removed, the NEC-violating radiation is also removed.

The Proof

- In the BPS case, only the $b_3\delta_{ij}$ term survives after modifying the stress tensor,

$$\langle \widetilde{T}_{0i}(x) D_j(y)\rangle = -b_3(1-\frac{1}{d})\frac{|x_\perp|^{p-d+2}}{|x-y|^{2p+4}}(x^0-y^0)\delta_{ij} \ . \tag{27}$$

  The kinematic structure comes from boosting the static field.

- If we choose a complex coordinate $z = x^1 + \mathrm{i}x^2$ in the transverse direction, then the BPS equation is equivalent to the following statement,

$$\langle \widetilde{T}_{0z}(x) D_z(y)\rangle = 0 \ , \qquad \langle T_{0z}(x) D_z(y)\rangle \propto \partial_0\partial_z \langle O(x) D_z(y)\rangle \ , \tag{28}$$

  or the following (more convenient for the proof),

$$\langle T_{zz}(x) D_z(y)\rangle \propto \partial_z^2 \langle O(x) D_z(y)\rangle \propto \partial_z^2\left(\frac{\bar{z}|x_\perp|^{p-d+2}}{|x-y|^{2p+2}}\right) \ . \tag{29}$$

- Note that $\partial_z\langle j_z(x) D_z(y)\rangle$ (non-zero in codimension-2) also has this kinematic structure. We will prove that $\langle T_{zz} D_z\rangle$ is a linear combination of $\partial_z^2\langle \Phi D_z\rangle$ and $\partial_z\langle j_z D_z\rangle$, where Φ and j are the scalar superprimary (if there is one) and R-symmetry current in the stress tensor multiplet,

$$(\Phi \xrightarrow{Q} \chi_\alpha \xrightarrow{Q}) \quad j_\mu \xrightarrow{Q} J_{\mu\alpha} \xrightarrow{Q} T_{\mu\nu} \tag{30}$$

- The strategy is to use superconformal Ward identities to relate $\langle TD\rangle$ with $\langle jD\rangle$ and $\langle \Phi D\rangle$. For example, $0 = Q\langle JD\rangle \sim \langle TD\rangle + \partial\langle jD\rangle + \partial_{\|}\langle J\Lambda\rangle$.

The Proof

- For given dimension d and p, it suffices to consider the least $\mathcal{N}$ which admits superconformal p-dimensional (rotation-invariant) defects.

defect	$(d,\mathcal{N})$	superalgebra	p-embedding
line	(3,2)	$\mathfrak{osp}(2\|4;\mathbb{R})$	$\mathfrak{su}(1,1\|1)\oplus\mathfrak{u}(1)_c$
	(4,2)	$\mathfrak{su}(2,2\|2)$	$\mathfrak{osp}(4^*\|2)$
	(5,1)	$F(4;2)$	$D(2,1;2;0)\oplus\mathfrak{su}(2)_c$
surface	(4,1)	$\mathfrak{su}(2,2\|1)$	$\mathfrak{su}(1,1\|1)\oplus\mathfrak{su}(1,1)_c\oplus\mathfrak{u}(1)_c$
	(5,1)	$F(4;2)$	$D(2,1;2;0)\oplus\mathfrak{so}(2,1)_c$
	(6,1)	$\mathfrak{osp}(8^*\|2)$	$\mathfrak{osp}(4^*\|2)\oplus\mathfrak{so}(2,1)_c\oplus\mathfrak{so}(3)_c$
$p=3$	(5,1)	$F(4;2)$	$\mathfrak{osp}(2\|4;\mathbb{R})\oplus\mathfrak{u}(1)_c$
$p=4$	(6,1)	$\mathfrak{osp}(8^*\|2)$	$\mathfrak{su}(2,2\|1)\oplus\mathfrak{u}(1)_c$

As a by-product, we obtain this classification of minimal embeddings. We examined the possible superalgebra embeddings using the methods of...

(D'Hoker, Estes, Gutperle, Krym, Sorba '08; Gutperke, Kaldi, Raaj '17; Agmon, Wang '20)

- For all cases apart from $(d,\mathcal{N})=(3,2),(4,1)$, there are scalars in the stress tensor multiplet that can be used in the modification of $T^{\mu\nu}$. However, our proof is purely algebraic and does not use the existence of such a scalar.
- The cases for $d=4$ and surfaces in $d=6$, $\mathcal{N}=(2,0)$ are already discussed in the literature. (Bianchi, Lemos, Meineri '18; Bianchi, Lemos '19; Drukker, Probst, Trépanier '20)

The Proof

- As a trick to unify the proofs, we make a suitable choice of the γ-matrix basis and of the defect orientation. The transverse complex coordinate is $z = x^1 + \mathrm{i}x^2$.

d	p	defect directions	preserved supercharges
6	2	3,4	Q^1_1, Q^1_3, Q^2_1, Q^2_3
	4	3,4,5,6	Q^1_1, Q^1_2, Q^2_3, Q^2_4
5	1	5	Q^1_1, Q^1_4, Q^2_1, Q^2_4
	2	3,4	Q^1_1, Q^1_3, Q^2_1, Q^2_3
	3	3,4,5	Q^1_1, Q^1_2, Q^2_3, Q^2_4
4	1	4	$\mathrm{Re}\{\mathcal{Q}^1_1, \mathcal{Q}^1_2, \mathcal{Q}^2_1, \mathcal{Q}^2_2\}$
	2	3,4	Q_1, $\bar{Q}_2$
3	1	3	Q_1, $\bar{Q}_2$

R-sym
$\downarrow$
Q^A_α
$\uparrow$
spinor

- Our proof only uses the first conserved supercharges in each case.

The Proof

- As an example, let us consider the case $d = 5$ for arbitrary p. The superconformal algebra is $F(4;2)$. The proof for the other dimensions is the same up to minor modifications.
- The $(32+32)$ stress tensor multiplet contains a scalar Φ, a spinor χ^A_α, a 2-form $B_{\mu\nu}$, the $\mathfrak{su}(2)$ R-symmetry currents j^I_μ, the supercurrents $J^A_{\mu\alpha}$ and $T_{\mu\nu}$,

$$\Phi \xrightarrow{Q} \chi^A_\alpha \xrightarrow{Q} j^I_\mu \oplus B_{\mu\nu} \xrightarrow{Q} J^A_{\mu\alpha} \xrightarrow{Q} T_{\mu\nu} \ . \tag{31}$$

(Córdova, Dumitrescu, Intriligator '16)

- The full transformation law of the stress tensor multiplet is

$$\begin{aligned}
Q^A_\alpha(\Phi) &= \chi^A_\alpha \ , \\
Q^A_\alpha(\chi^B_\beta) &= j^I_\mu\,(\gamma^\mu)_{\alpha\beta}(\sigma_I)^{AB} + B_{\mu\nu}\,(\gamma^{\mu\nu})_{\alpha\beta}\,\varepsilon^{AB} + (\partial_\mu\Phi)\,(\gamma^\mu)_{\alpha\beta}\,\varepsilon^{AB} \ , \\
Q^A_\alpha(j^I_\mu) &= \frac{1}{2}J^B_{\mu\alpha}\,(\sigma^I)_B{}^A - \frac{1}{4}(\partial_\nu\chi^B_\beta)\,(\sigma^I)_B{}^A(\gamma_\mu{}^\nu)_\alpha{}^\beta \ , \\
Q^A_\alpha(B_{\mu\nu}) &= -\frac{1}{4}J^A_{\rho\beta}\,(\gamma_{\mu\nu}{}^\rho)_\alpha{}^\beta + \frac{1}{8}(\partial_\rho\chi^A_\beta)\,(2\,\gamma_{\mu\nu}{}^\rho + \delta_{[\mu}{}^\rho\gamma_{\nu]})_\alpha{}^\beta \ , \\
Q^A_\alpha(J^B_{\mu\beta}) &= 2\,T_{\mu\nu}\,(\gamma^\nu)_{\alpha\beta}\,\varepsilon^{AB} - \frac{1}{2}(\partial_\nu j^I_\rho)\,(\gamma_\mu{}^{\nu\rho} - 3\,\delta_\mu{}^\rho\gamma^\nu)_{\alpha\beta}(\sigma_I)^{AB} \\
&\quad -\frac{1}{2}(\partial_\nu B_{\rho\sigma})\,(\gamma_\mu{}^{\nu\rho\sigma} + 2\,\delta_\mu{}^\rho\gamma^{\nu\sigma} - 2\,\eta^{\nu\rho}\gamma_\mu{}^\sigma + 6\,\delta_\mu{}^\rho\eta^{\nu\sigma})_{\alpha\beta}\,\varepsilon^{AB} \ , \\
Q^A_\alpha(T_{\mu\nu}) &= \frac{1}{4}(\partial_\rho J^A_{\mu\beta})\,(\gamma^\rho{}_\nu)_\alpha{}^\beta + (\mu\leftrightarrow\nu) \ .
\end{aligned} \tag{32}$$

The details of the derivation can be found in our paper. (Bachas, Bianchi, ZC '25)

The Proof

$$\Phi \xrightarrow{Q} \chi_\alpha^A \xrightarrow{Q} j_\mu^I \oplus B_{\mu\nu} \xrightarrow{Q} J_{\mu\alpha}^A \xrightarrow{Q} T_{\mu\nu} \ . \tag{33}$$

- The action of $Q = Q_1^1$ on the fields is simple,

$$Q(T_{zz}) = \tfrac{1}{2}\partial_z J_{z1}^1 \ , \quad Q(j_z^3) = \tfrac{1}{2} J_{z1}^1 + \tfrac{1}{4}\,\partial_z \chi_1^1 \ , \quad Q(\Phi) = \chi_1^1 \ . \tag{34}$$

- There exists a fermionic defect operator Λ (a 'displacino') such that $Q(\Lambda) = D_z$ (without a derivative of the scalar partner, the 'tilt').
- We will use the following superconformal Ward identities associated to $Q = Q_1^1$,

$$\begin{aligned} 0 &= Q\langle T_{zz}(x)\Lambda(y)\rangle = \frac{1}{2}\partial_z\langle J_{z1}^1(x)\Lambda(y)\rangle + \langle T_{zz}(x)D_z(y)\rangle \ , \\ 0 &= Q\langle j_z^3(x)\Lambda(y)\rangle = \frac{1}{2}\langle J_{z1}^1(x)\Lambda(y)\rangle + \frac{1}{4}\partial_z\langle \chi_1^1(x)\Lambda(y)\rangle + \langle j_z^3(x)D_z(y)\rangle \ , \\ 0 &= Q\langle \Phi(x)\Lambda(y)\rangle = \langle \chi_1^1(x)\Lambda(y)\rangle + \langle \Phi(x)D_z(y)\rangle \ . \end{aligned} \tag{35}$$

- As promised, we find that $\langle T_{zz}D_z\rangle$ can be expressed as a linear combination of $\partial_z\langle j_z D_z\rangle$ and $\partial_z^2\langle \Phi D_z\rangle$,

$$\langle T_{zz}(x)D_z(y)\rangle = \partial_z\langle j_z^3(x)D_z(y)\rangle - \frac{1}{4}\partial_z^2\langle \Phi(x)D_z(y)\rangle \propto \partial_z^2\left(\frac{\bar{z}|x_\perp|^{p-d+2}}{|x-y|^{2p+2}}\right) . \tag{36}$$

Q.E.D.

Conclusion

- Supersymmetry leads to a linear relation between the energy stored in the defect and its stiffness.
- Supersymmetry ensures that restoring NEC also removes the transverse stress.
- Graham–Witten anomalies, e.g. for $p = 2$ defects,

$$T_\mu{}^\mu\big|_{\text{defect}} = \frac{1}{24\pi}\left(a^{(2)} R + d_1^{(2)} \widetilde{K}^i_{ab} \widetilde{K}_i^{ab} - d_2^{(2)} W_{ab}{}^{ab}\right) , \qquad d_1^{(2)} \propto C_D \ , \qquad d_2^{(2)} \propto a_T \ . \tag{37}$$

(Graham, Reichert '17; Chalabi, Herzog, O'Bannon, Robinson, Sisti '21)

- Schott term in rediation-reaction force.
- Higher order defect Witten diagrams.

Thank you for your attention!